



Research paper

Multi-dimensional vibration control for offshore floating platform synergizing built-in wave energy converter with decoupled power take-offs

Yongxing Zhang^a, Zhicong Huang^{a,*}, Jing Bian^b

^a Shien-Ming Wu School of Intelligent Engineering, South China University of Technology, Guangzhou, 511442, China

^b School of Civil Engineering, Chongqing University, Chongqing, 400044, China

ARTICLE INFO

Keywords:

Offshore floating platform
Decoupled power take-offs
Vibration control
Wave energy converter
Energy harvesting

ABSTRACT

The hybrid system of the offshore floating platform and wave energy converter provide a potential solution for achieve floating platform's vibration control by energy harvesting. However, existing hybrid systems still face significant challenges in maintaining effective multi-dimensional vibration control under changing wave frequencies and directions. To bridge this research gap, this paper proposes a novel floating platform synergizing built-in wave energy converter with decoupled power take-offs. The built-in configuration isolates the wave energy converter from ocean waves, thereby eliminating the hydrodynamic effects acting on the wave energy converter. Meanwhile, the decoupled design enables that the operating axes of power take-offs in wave energy converter align one-to-one with vibration directions of floating platform. These features allow the vibrations in different directions of the floating platform to be directly harvested by the WEC and converted into electrical energy. A classic Spar floating platform is used as an example, and the entire system of Spar synergizing built-in wave energy converter with decoupled power take-offs is designed. A multi-physical domain mathematical model is built and validated. Based on this model, an optimization framework oriented to multi-dimensional vibration control is proposed. The optimization results are analyzed to demonstrate that the proposed structure exhibits excellent multi-dimensional vibration control ability through energy harvesting under varying wave states.

1. Introduction

The ocean, as the cradle of life, harbors vast energy resources, encompassing both traditional fossil fuels and various renewable forms of energy (Oliveira-Pinto et al., 2019; Pata, 2021; Zhao et al., 2024). In recent years, substantial energy extraction has occurred in near-shore regions. However, the focus of offshore energy extraction is increasingly shifting to deeper waters due to the depletion of shallow water resources and advancements in technology (Rehman et al., 2023; Sharma et al., 2010). Offshore floating platforms (OFPs) are critical systems for accessing and exploiting these deep-water resources. These platforms support a range of installations, including fossil fuel extraction facilities, wind power systems, wave energy converters (WECs) and power distribution stations. For this reason, OFPs have become a focal point of interest for numerous coastal nations (Lamas-Pardo et al., 2015; Saeed et al., 2024).

Various types of OFPs have been investigated and developed in recent decades, including the semi-submersible floating platform

(Sharma et al., 2010), Spar floating platform (Saeed et al., 2024), and tension leg floating platform (Lamas-Pardo et al., 2015). These platforms are constructed in different structural configuration modes to meet their own specific operation requirements and purposes. However, in deep ocean environments, these platforms are inevitably subjected to ocean waves, causing them to vibrate in multiple motion directions. Under some extreme wave conditions, excessive vibrations can significantly impact OFP performance and crew comfort, and may also increase the risk of fatigue in structural components, possibly leading to overturning or sinking. On the other hand, reducing the vibration amplitude of the OFP by 15% could more than double its operational lifespan and substantially reduce maintenance and inspection costs (Zhang et al., 2017; Ou et al., 2007). Consequently, how to reduce the vibration of OFPs has become the main concern for relative scholars and companies in this field.

Numerous structural control methods from the field of civil engineering have been introduced and investigated to mitigate the vibrations of OFPs. Increasing stiffness represents a most direct and simple approach, which can shift the OFP's resonance frequency away from the

* Corresponding author.

E-mail addresses: wicmyazzhangyx@mail.scut.edu.cn (Y. Zhang), zhiconghuang@scut.edu.cn (Z. Huang).

<https://doi.org/10.1016/j.oceaneng.2025.120450>

Received 9 November 2024; Received in revised form 1 January 2025; Accepted 18 January 2025

Available online 27 January 2025

0029-8018/© 2025 Elsevier Ltd. All rights are reserved, including those for text and data mining, AI training, and similar technologies.

Abbreviations

OFF	Offshore floating platform
WEC	Wave energy converter
PTO	Power take-off
NREL	National Renewable Energy Laboratory
WECDP	WEC with decoupled PTOs
Spar-WECDP	Spar synergizing built-in WECDP
RAO	Response amplitude operator

excitation frequency range by enhancing its stiffness (Ma et al., 2023; Kandasamy et al., 2016). Nevertheless, this method inevitably escalates construction expenses and alters the existing OFP design. Accordingly, another approach is to apply the structural control methods to minimize platform vibrations (Zhang et al., 2017; Yao, 1972; Hemmati et al., 2019; Yang et al., 2019). Current structural vibration control methods can be classified into three main categories: passive, active, and semi-active controls (Chai et al., 2024; Farahpour and Hejazi, 2023; Huang and Yang, 2023). Semi-active and active control methods offer effective vibration control by providing feedback control force. However, these methods require external energy to maintain their control force requirements. By contrast, the control force of passive methods is derived from the motion of the devices themselves and does not require any external energy.

Passive vibration control techniques primarily involve energy consumption (e.g., dampers (Ma et al., 2021; Ma et al., 2019)), energy isolation (e.g., isolators/isolation platforms (Bian et al., 2023; Ma et al., 2020; Ke et al., 2023)), and vibration transfer (e.g., dynamic vibration absorbers (Su et al., 2023a; Su et al., 2024; Su et al., 2023b; Bian and Jing, 2020)). While these reported vibration control devices can decrease the OFPs' vibration amplitude, they belong to passive approaches and suffers from energy dissipation, which means that the wave energy captured by the OFP is totally wasted. Meanwhile, OFP requires energy sources to support living facilities, operate equipment, and power scientific research instruments. To this end, this situation motivates us to design an OFP that can suppress the vibration by absorbing the wave energy and regenerating it as useful electrical energy.

The combination of OFP and wave energy converter (WEC) has been explored to concurrently achieve vibration control and energy harvesting (Zhang et al., 2024a; Zhang, 2023; Rony and Karmakar, 2022; Nguyen and Wang, 2020). This hybrid system employs multiple WECs placed outside the OFP to reduce wave loads or introduce additional restoring forces and positive damping (Si et al., 2021; Aboutalebi et al., 2024), thereby decreasing platform vibration amplitude. For hybrid WEC-floating platform systems, some studies with significant contributions have been conducted in numerical investigations. For instance, Cheng et al. (2022) investigated a hybrid WEC-breakwater system combining an oscillating water column and an oscillating buoy, demonstrating superior performance in both wave energy harvesting and wave attenuation compared to standalone devices. To gain a deeper understanding of such hybrid systems, Cheng et al. (Cheng, 2024) also explored a single-point moored offshore cage-WEC hybrid system, highlighting its efficient wave energy capture under varying wave conditions and its hydrodynamic coupling effects. Furthermore, experiments on a dual-pontoon WEC hybrid system (Cheng, 2022a) highlighted its enhanced energy harvesting and wave damping capabilities. Other prototype experiments have also shown in some studies (Zhu and Hu, 2016; Gaspar et al., 2018; Kamarlouei et al., 2020) that employing WECs around the OFP can suppress platform vibration to a certain extent. Nonetheless, these prototype experiments are limited to motion response analysis under fixed wave states, neglecting the dynamic coupling between the OFP and WEC in changing wave states. Further

studies (Zhang et al., 2023a, 2023b; Zhou et al., 2023; Tian et al., 2023) reveals that the vibration control performance of this hybrid system is unsatisfactory under varying wave frequencies and directions. The issues are mainly reflected in the following two aspects: (1) Vibration suppression of the OFP is only effective within a limited wave frequency range. Outside this range, shifts in wave frequencies can alter hydrodynamic effects—such as hydrostatic stiffness and radiation damping—resulting in an unexpected increase in platform vibration amplitude. (2) Even at a specific wave frequency, the current hybrid systems do not ensure effective vibration control in all vibration directions of the OFP. When the wave direction changes, this issue becomes trickier due to the increased complexity of the OFP vibrations.

The key to the first issue is to eliminate the negative influence of the WEC's hydrodynamic effects on the OFP as wave frequency varies. In view of this situation, a variety of studies have been undertaken to improve WEC's hydrodynamics through structural parameter optimization, such as optimizing the WEC shape (Wang et al., 2022; Wu et al., 2024), power take-off (PTO) parameters (Si et al., 2021; Zhou et al., 2023), and the number of WECs (Tian et al., 2023; Hu et al., 2020; Ghafari et al., 2021). However, the WECs in these methods are still deployed outside the OFP. This notion means that, despite improvements, the hydrodynamic effects exerted by ocean waves on the WEC cannot be entirely eliminated. Therefore, the optimized structural parameters of the WEC based on hydrodynamics will not be necessarily guaranteed to meet the preset vibration control requirements in the random ocean wave frequencies.

The core of the second issue is to enable the PTO of the WEC to harvest the vibration energy of the OFP in different directions. Cheng (2022b) innovatively proposed deploying an oscillating wave surge converter and two oscillating heave converters outside the OFP. With the surge and heave PTOs in the WEC, the OFP's vibration energy in the surge and heave directions is harvested, thereby suppressing vibrations in these two directions. However, this scheme does not fully eliminate the hydrodynamic effects exerted by ocean waves on the WEC, which may result in unreliable vibration control under varying wave frequencies. More importantly, the operating axes of the PTOs no longer align with the vibration direction of the OFP as the wave direction changes, making it difficult to maintain effective vibration control through energy harvesting.

Based on the above analysis, achieving effective multi-dimensional vibration control requires the PTOs' operating axes to align with the OFP's vibration direction, while eliminating the hydrodynamic effects acting on the WEC. In our recently published work (Huang et al., 2024), we proposed using a built-in heave WEC for offshore floating platform vibration control. However, this approach only addressed vibration control in the heave direction, neglecting the need for vibration control in other motion directions. In contrast, this study introduces a novel built-in WEC with decoupled power take-offs, enabling multi-dimensional vibration control of the floating platform. This approach broadens the application of built-in WEC integrated with floating platforms, making it more suitable for vibration control in complex ocean environments. Furthermore, this study conducts a more comprehensive parameter analysis to explore the feasibility of vibration control and energy harvesting across multiple DOF directions. This analysis provides insights into the WEC's energy harvesting and the Spar's motion in different DOF directions, contributing to a deeper understanding of the importance of each DOF in the overall system performance.

In our proposed concept, the decoupled design ensures that multi-PTOs in the WEC align one-to-one with the various vibration directions of the OFP, and the built-in configuration eliminates the hydrodynamic effects on the WEC caused by ocean waves. These features allow the vibrations in different directions of the OFP to be directly harvested by the WEC and converted into electrical energy. Without loss of generality, a classic Spar floating platform is used as an example of OFP. The design concept of the Spar synergizing built-in WECDP (Spar-

WECDP) is presented in Fig. 1. The entire Spar-WECDP system, including the mechanical and energy harvesting structures, is proposed following this idea. Subsequently, a detailed mathematical model that incorporates multiple physical domains is built and validated. Based on this model, the optimization analysis oriented to multi-dimensional vibration control is conducted. The key contributions of this study are outlined below:

- (1) A novel WEC with decoupled power take-offs is proposed for the vibration control of the OFP, providing a viable solution for achieving multi-dimensional vibration control while simultaneously harvesting energy.
- (2) A mathematical model of Spar-WECDP integrating hydrodynamic, mechanical, and electrical aspects, is developed and validated, providing guidance for modelling this type of multi-physical domain coupling.
- (3) An optimization framework oriented to multi-dimensional vibration control is developed, analyzing the impact on the vibration control and energy harvesting performance of the Spar-MPDWEC under varying wave frequencies, wave directions, and mass ratios.

The paper is structured as follows: Section 2 provides a detailed explanation of the mechanical design, energy harvesting structure, and operational mechanism of the Spar-WECDP. Section 3 establishes the multi-physical domain mathematical models including mechanical, hydrodynamic and electric. Section 4 conducts the performance optimization and analysis oriented to multiple dimensional vibration control. Finally, Section 6 provides the conclusion of the paper.

2. Structural design and description

The detail structure design scheme of the Spar-WECDP is shown in Fig. 2. As shown in Fig. 2, the WECDP is internally installed into the Spar. The Spar's protective barrier creates a secure seal against water

ingress, protecting the WECDP's internal components. More importantly, this internal installation structure can isolate the WECDP from ocean waves, preventing the induction of radiation-damping and hydrostatic stiffness effects on the WECDP, thus allowing the WECDP to directly harvest Spar's vibration energy. The proposed WECDP system has five DOFs: surge, sway, heave, pitch, and roll. The yaw motion is neglected in this study due to the axisymmetric cylindrical structure of the Spar (Wei-xing et al., 2018; Chen, 2021). In Fig. 2, this structure is categorized into two parts: a 2-DOF structure (surge and sway) and a 3-DOF structure (heave, pitch, and roll). These two parts are connected by a top frame marked in yellow.

2.1. Structural feature

The first is the detail description for the 2-DOF structure. This portion includes two prismatic joints corresponding to the surge and sway directions. To drive the rotational generator, it is necessary to design a corresponding translational PTO that converts linear motion into rotational motion. As is well known, ball screw can convert the linear motion of the nut into the rotational motion of the screw. Utilizing this characteristic, the ball screw's output shaft is connected to the generator's input shaft via a coupler, forming a translational PTO. As shown in Fig. 3, four translational PTOs are symmetrically arranged in an "I-shaped" configuration, aligned with the horizontal x and y axes. This configuration enables the proposed WEC to achieve surge and sway motions and drive the generator to rotate by the translational PTOs.

The second is the detail description for the 3-DOF mechanical structure. As shown in Fig. 4, this structure follows a typical parallel configuration, consisting of a passive linkage and four active linkages. These linkages connect the oscillating body to the top frame. In this configuration, the passive linkage imposes all necessary motion constraints on the end-effector (i.e. oscillating body), thereby determining the structure's DOFs (Zhang, 2023; Kuo and Dai, 2013). In contrast, the active linkage functions as an unconstrained branch, ensuring that the end effector can achieve the required motion DOFs (Zhang, 2023; Lin

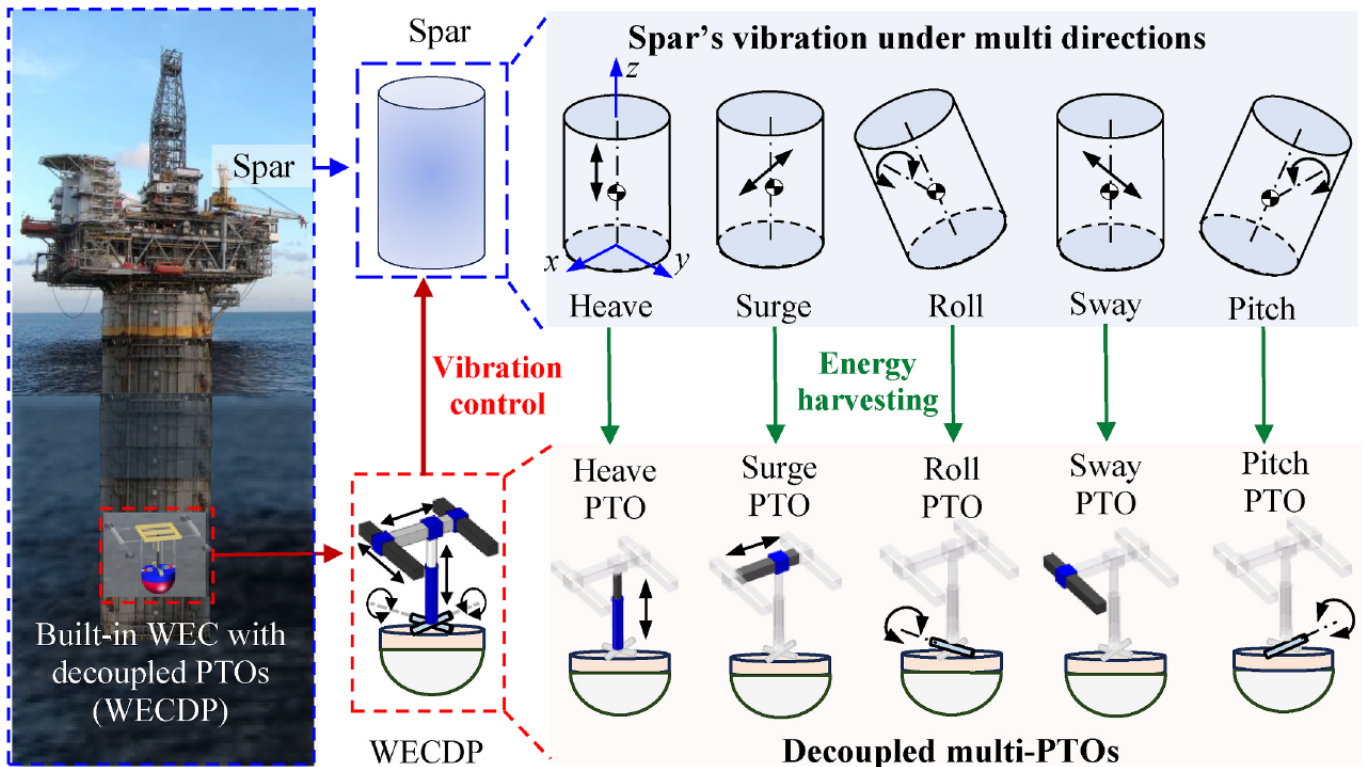


Fig. 1. Conceptual design of the Spar synergizing built-in WECDP for achieving multi-dimensional vibration control.

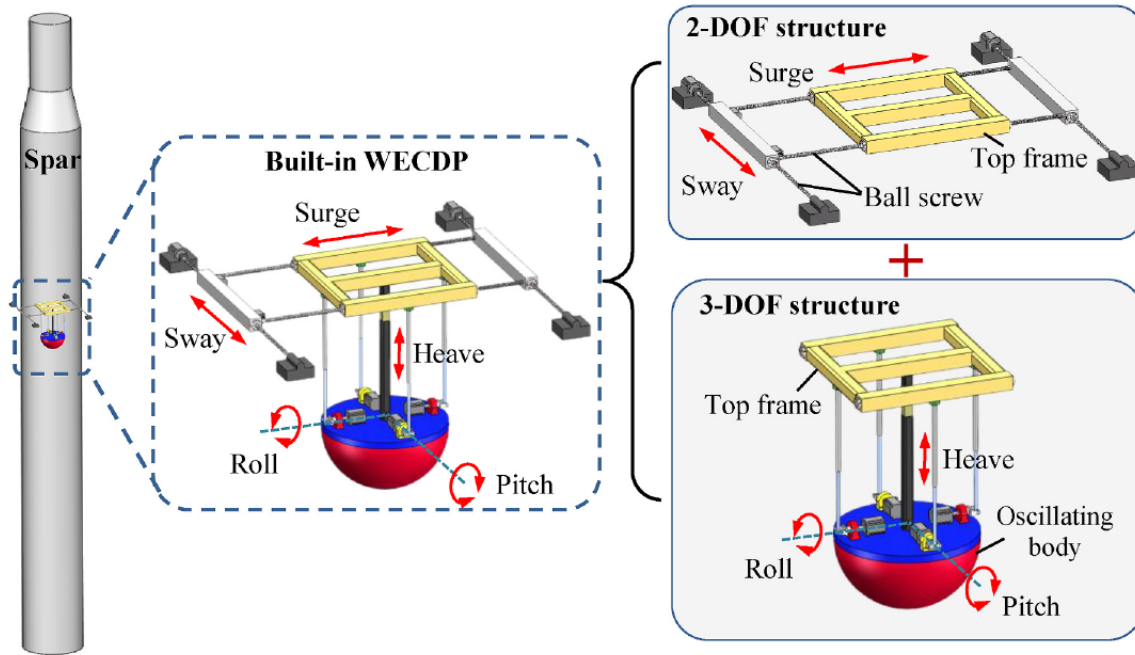


Fig. 2. Mechanical and energy harvesting structure design scheme of Spar-WECDP.

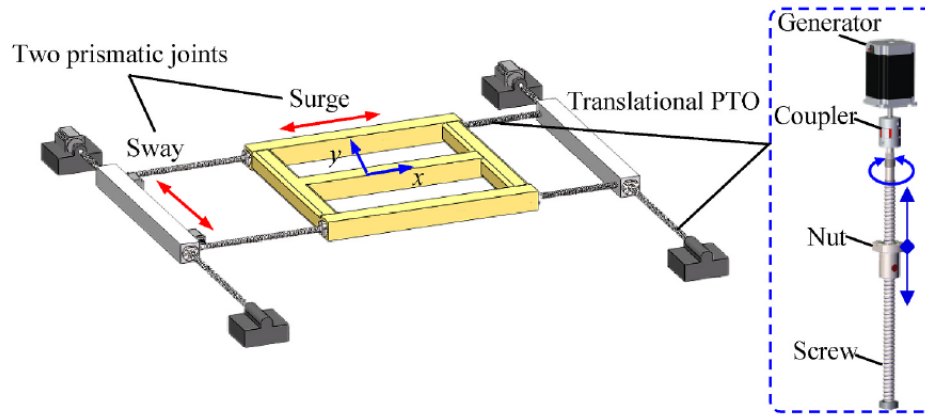


Fig. 3. The detail description for the 2-DOF mechanical structure.

et al., 2022).

Regarding the passive linkage: According to the above linkage definition, the passive linkage is designed by combining a vertical universal joint (allowing roll and pitch) and a prismatic joint (allowing heave), thus meeting the 3-DOF structure requirements. As shown in Fig. 5, this passive linkage connects the oscillating body to the top frame, enabling the oscillating body to achieve rotational motion in the roll and pitch directions as well as heave motion.

Regarding the active linkage: As shown in Fig. 6, each active linkage consists of a spherical joint, a prismatic joint, and a horizontal universal joint, ensuring no interference with the heave, roll, and pitch motions allowed by the passive linkage. This unconstrained design is widely used in the field of parallel structures (Zhang, 2023; Zhao et al., 2020). Please note that, unlike the vertical universal joint mentioned earlier, the horizontal universal joints in the active linkages provide a rotational shaft that can connect to the generator's input shaft. This establishes a structural foundation for converting the oscillating body's rotational motion into the generator's rotational motion.

As shown in Fig. 6, with all linkages installed, the oscillating body can still effectively achieve rotational motion in the roll and pitch directions as well as heave motion. The horizontal rotational shaft of the

universal joint connects sequentially to the gearbox (for speed amplification (Zhang et al., 2024b; Zhou et al., 2024)) and the generator, enabling the conversion of the oscillating body's rotational motion into the generator's rotational motion.

2.2. Motion description

The above description highlights the mechanical features of the proposed WEC system. Figs. 7 and 8 further illustrates how the rotational and translational motions occur automatically. As shown in Fig. 7, the WEC is initially installed horizontally inside the Spar. When the Spar experiences pitch and roll motions due to wave excitation, the WEC also rotates along with the Spar due to its built-in installation mode. However, driven by the inertia of the oscillating body, it tends to maintain its original state, thereby automatically generating rotational motion.

Similarly, when the Spar undergoes heave, surge, and sway motions under wave excitation, the proposed WEC also moves along with the Spar due to its built-in installation mode. Driven by the inertia of the oscillating body, it tends to maintain its original state, thereby automatically generating translational motion. Please note that, due to the axisymmetric of proposed structure, the sway motion is similar to the

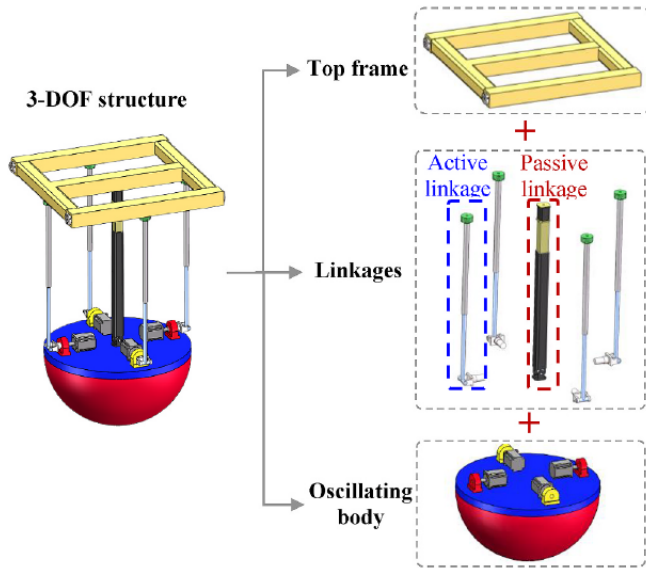


Fig. 4. The detail description for the 3-DOF mechanical structure.

surge motion and is not detailed in Fig. 8 for brevity.

2.3. System compatible design

To better align the research results with practical applications, this study adopts the structural data of the real Spar floating platform provided by the National Renewable Energy Laboratory (NREL) (Jonkman, 2010). These platform data are closer to commercial designs (see Table 1).

Secondly, to determine the structural data of the proposed built-in WECDP, it is necessary to first design it to be compatible with the above real Spar platform. The design process involves determining the built-in WEC system's mass, materials, and geometry shape.

- Regarding the mass: In the field of mechanical vibration control, a mass ratio below 5% is widely recognized as a practical upper limit to minimize costs and save space (Su, 2023; Anh, 2013; Park, 2022). Following this principle, the mass of the built-in WEC with vibration damping capabilities is set at 5% of the Spar's mass (i.e., 373.3 tons).

This 5% mass ratio provides a clear reference point that is useful for subsequent analysis and performance evaluation.

- Regarding the materials: To reduce construction cost, the widely available and cost-effective stainless steel (density: $8 \times 10^3 \text{ kg/m}^3$) is adopted for built-in WEC systems. Meanwhile, its excellent corrosion resistance in humid environments makes it a more reasonable choice for offshore applications.
- Regarding the geometry shape: To enhance practicality, this study adopts a hemispherical shape which is commonly used in the WEC field. This shape not only facilitates production for current manufacturers but also reduces construction complexity.

After completing the compatibility design of the system, the built-in WEC is modeled using 3-D modeling software SolidWorks based on the above-defined parameters. The material properties are uniformly distributed throughout the model. At the initial installation, the WEC is installed at the same height as the Spar's mass center. Finally, using the calculation modules within the modeling software, the system parameters of the built-in WEC can be obtained as follows:

Furthermore, to provide readers with a clearer understanding of the proposed structure, Fig. 9 illustrates the layout of the Spar-WECDP system. Positioned in 320 m deep water, the Spar has a draft of 120 m and is anchored by mooring lines. The fixed coordinate system B is centered at the Spar's origin at mean sea level (MSL). The proposed structure is axially symmetric, with the Spar's overall height measuring 130 m and the distance from the Spar's mass center to its bottom being 30.085 m. At the initial installation, the WEC is positioned at the same height as the Spar's mass center. Tables 1 and 2 present the detailed structural parameters.

3. Mathematical modeling

To evaluate the proposed Spar-WECDP's performance, this section constructs its motion response model under regular and irregular waves according to linear potential theory (Cheng, 2022b; Tao et al., 2021; Bao et al., 2023) and the Newton-Euler. The hydrodynamic parameters of the Spar platform are analyzed using the commercial software ANSYS AQWA, which is based on the Boundary Element Method. Then, based on the operation mechanism of the energy harvesting structure, the dynamics including the permanent magnet synchronous generator (PMSG), power converter, and battery, are constructed to determine the output power of WECDP.

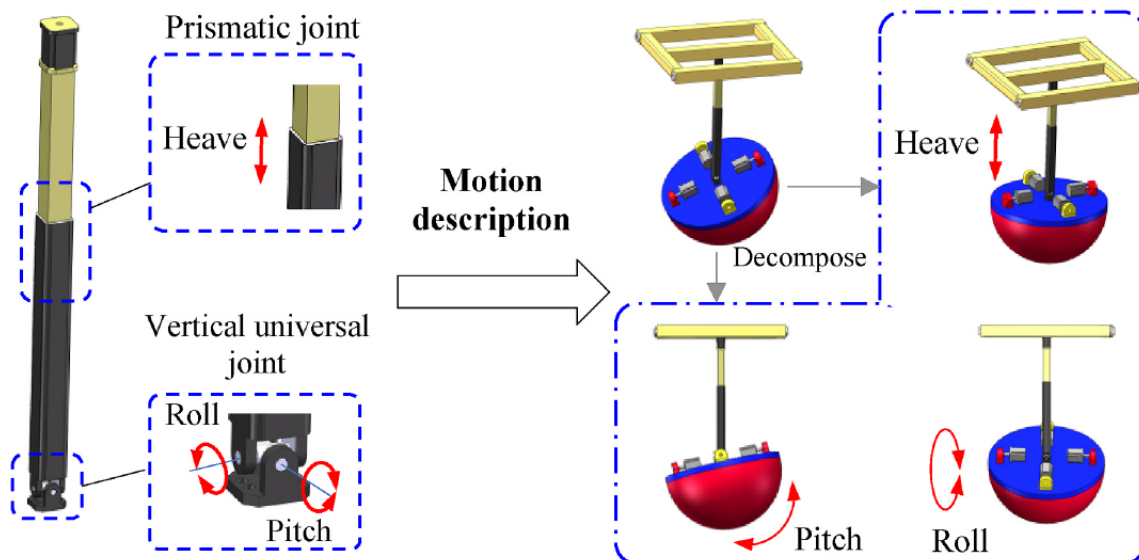


Fig. 5. The passive linkage and motion description for the 3-DOF mechanical structure.

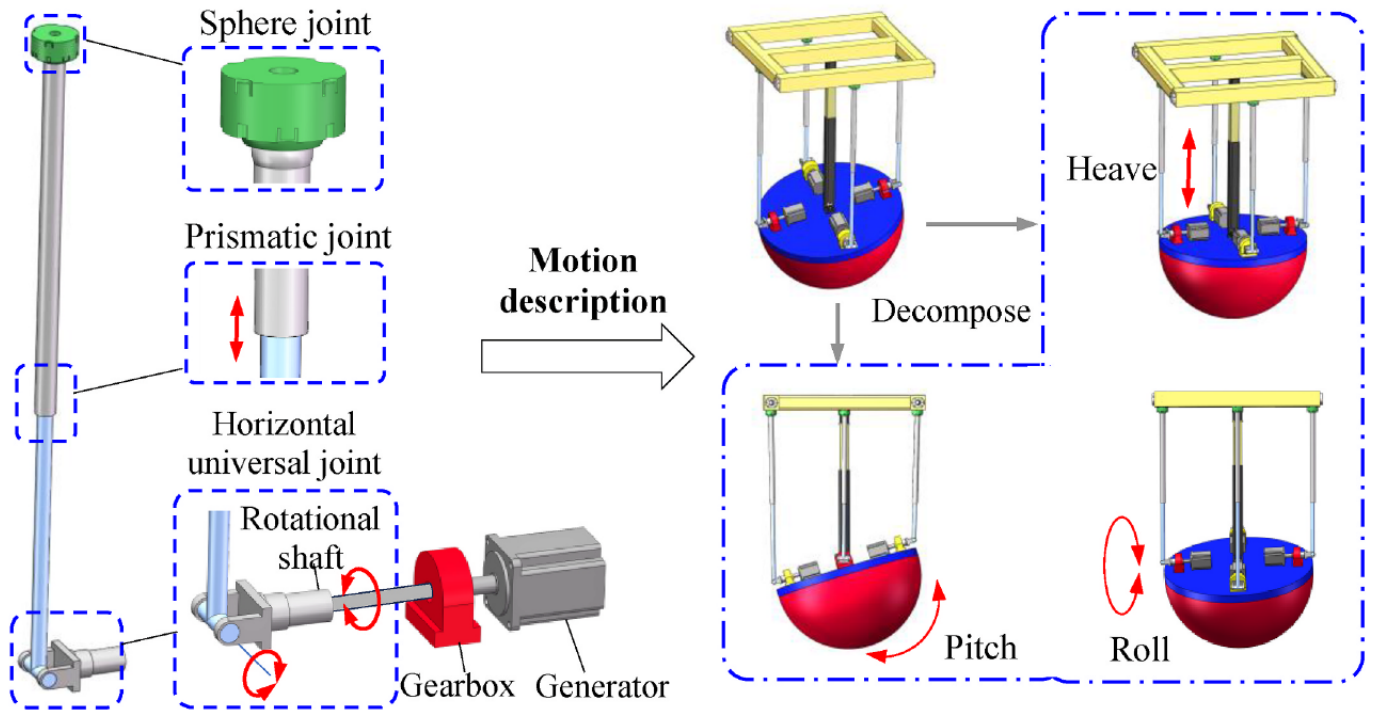


Fig. 6. The active linkage and motion description for the 3-DOFs mechanical structure.

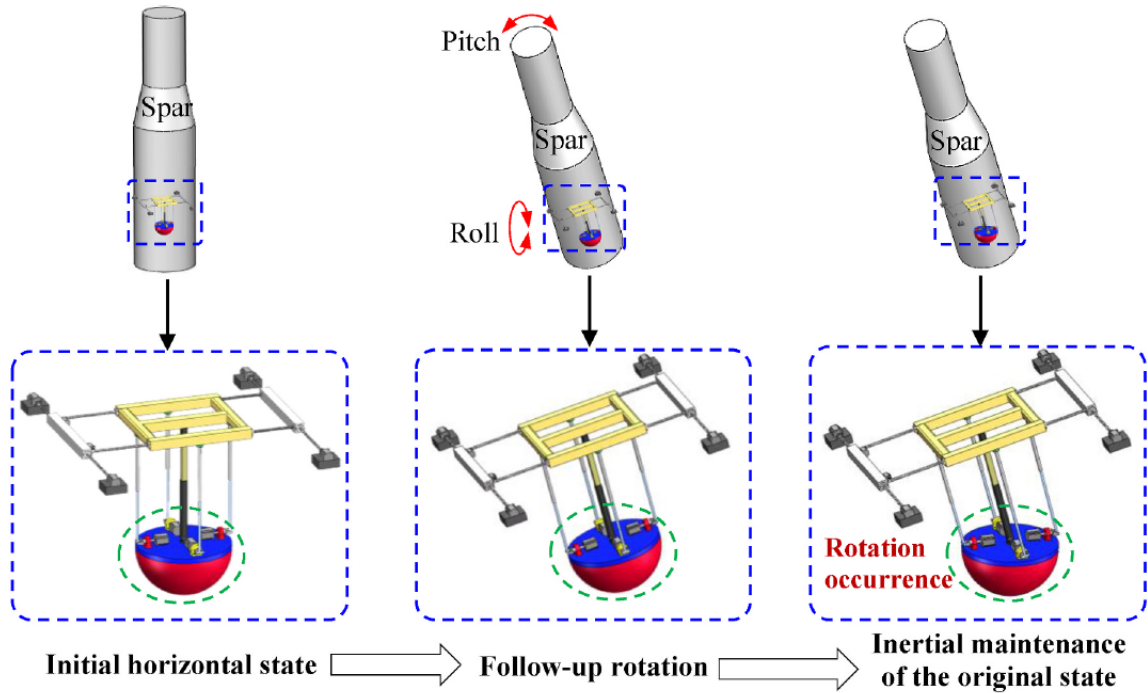


Fig. 7. Schematic diagram of the WECDP rotational motion occurring automatically.

3.1. Dynamic equations of Spar-WECDP

The coordinate frame of the Spar-WECDP is constructed as shown in Fig. 10. The xy plane of the fixed coordinate system $B-xyz$ is placed on the waterplane. The origin B of the coordinate system is at the center of the waterplane, the x -axis is aligned longitudinally along the Spar, the z -axis is oriented vertically upward, and the y -axis satisfies the right-hand rule. The $A-u_s v_s w_s$ is the moving coordinate system established with the Center of mass A of the Spar.

Under wave excitation, the Spar experiences rotation motion in the pitch and roll directions, while yaw rotation can generally be neglected due to the Spar's axisymmetric structure. As shown in Fig. 11, the Spar starts with its moving coordinate system $\{A\}$, where the u_s , v_s , and w_s axes of $\{A\}$ initially align with the x , y , and z axes of the fixed coordinate system $\{B\}$. Since translation does not affect rotation, the moving coordinate system $\{A\}$ is here defined to coincide with the fixed coordinate system $\{B\}$ for observing the rotation changes.

In Fig. 11, the Spar's moving coordinate system first rotates about the

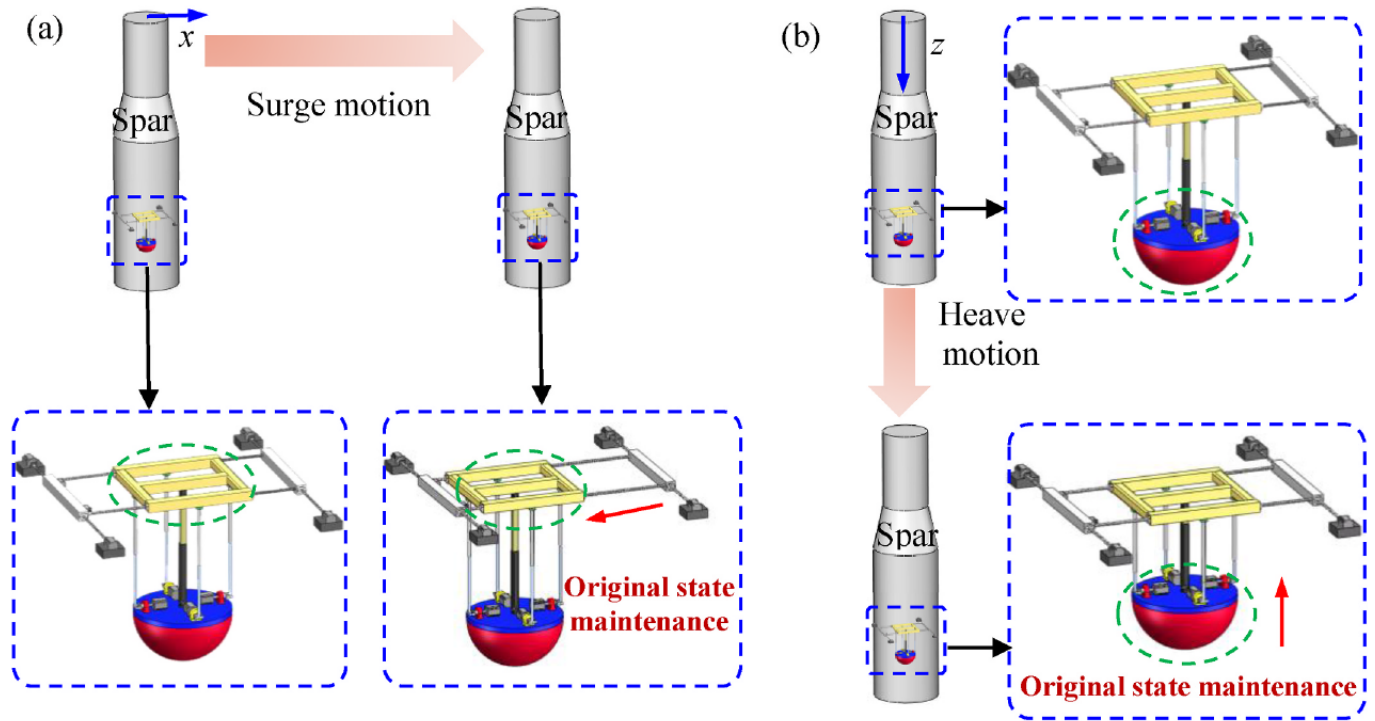


Fig. 8. Schematic diagram of the WECDP translational motion occurring automatically: (a) in the surge direction and (b) in the heave direction.

Table 1

Detailed specifications of the real Spar floating platform (Jonkman, 2010).

Configuration	Item	Symbol	Value
Spar	Operating draft	D_{ra}	120 m
	Overall height	H_{spar}	130 m
	Bottom diameter	R_s	9.4 m
	Distance from Spar mass center to bottom	$C_{G, Spar}$	30.085 m
	Physical mass	m_s	7466 t
	Roll inertia about Spar's mass center	$I_{xx, Spar}$	$4.229 \times 10^9 \text{ kg} \cdot \text{m}^2$
	Pitch inertia about Spar's mass center	$I_{yy, Spar}$	$4.229 \times 10^9 \text{ kg} \cdot \text{m}^2$
	Yaw inertia about Spar's mass center	$I_{zz, Spar}$	$1.642 \times 10^8 \text{ kg} \cdot \text{m}^2$
	Water depth	D_p	320 m

x -axis of the fixed coordinate system by an angle ψ_x . Then, based on the current state of Spar moving coordinate system $\{A\}$, it rotates about the y -axis of the fixed coordinate system by an angle ψ_y . In the above process, the rotation matrix ${}^B\mathbf{R}_A$ is used to describe the orientation of the Spar's moving coordinate system $\{A\}$ relative to the fixed coordinate system $\{B\}$. Since the above two rotations take place about the axes of a fixed coordinate system, the rotation matrix ${}^B\mathbf{R}_A$ can be obtained by sequentially left-multiplying the two basic rotation matrices $\mathbf{R}_{rot(x)}(\psi_x)$ and $\mathbf{R}_{rot(y)}(\psi_y)$ (Taghirad, 2013) as follows:

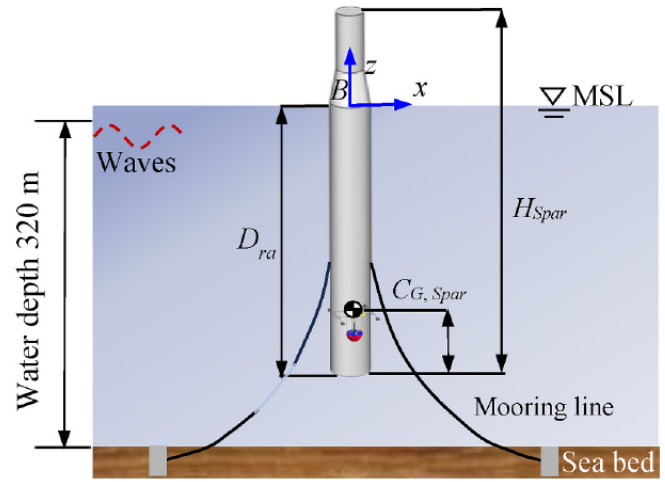


Fig. 9. Structural layout of the proposed Spar-WECDP.

Taking the Spar as the reference for the built-in WECDP, the coordinate system $\{A\}$ on the Spar can be treated as the fixed coordinate system of the built-in WECDP. Moreover, the built-in WECDP exhibits

$${}^B\mathbf{R}_A = \mathbf{R}_{rot(y)}(\psi_y) \mathbf{R}_{rot(x)}(\psi_x) = \begin{bmatrix} \cos \psi_y & 0 & \sin \psi_y \\ 0 & 1 & 0 \\ -\sin \psi_y & 0 & \cos \psi_y \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi_x & -\sin \psi_x \\ 0 & \sin \psi_x & \cos \psi_x \end{bmatrix} = \begin{bmatrix} \cos \psi_y & \sin \psi_x \sin \psi_y & \cos \psi_x \sin \psi_y \\ 0 & \cos \psi_x & -\sin \psi_x \\ -\sin \psi_y & \cos \psi_y \sin \psi_x & \cos \psi_x \cos \psi_y \end{bmatrix} \quad (1)$$

Table 2
Detailed specifications of the built-in WEC.

Configuration	Item	Symbol	Value
Built in WEC	Physical Mass	m_c	373.3 t
	Hemispherical radius	r_w	2.815 m
	Distance from WEC mass center to hemisphere top	$C_{G_s, WEC}$	1.06 m
	Roll inertia about WEC's mass center	$I_{xx, WEC}$	$7.666 \times 10^5 \text{ kg} \cdot \text{m}^2$
	Pitch inertia about WEC's mass center	$I_{yy, WEC}$	$7.666 \times 10^5 \text{ kg} \cdot \text{m}^2$
	Yaw inertia about WEC's mass center	$I_{zz, WEC}$	$1.182 \times 10^6 \text{ kg} \cdot \text{m}^2$

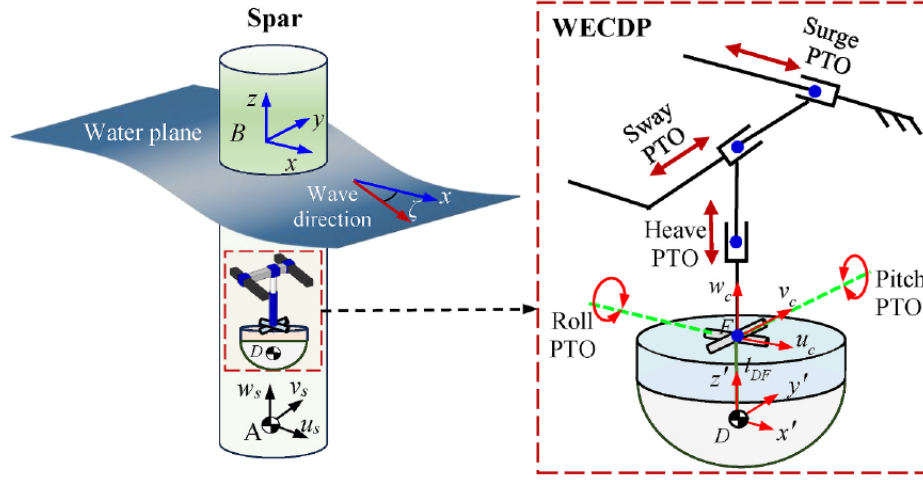


Fig. 10. Structure schematic of the Spar-WECDP.

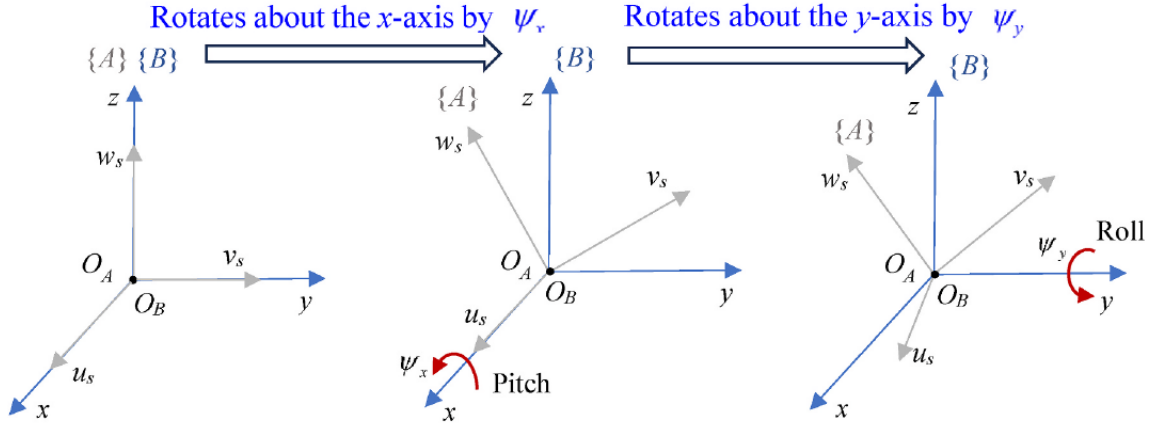


Fig. 11. The successive rotations of Spar about the fixed coordinate system.

rotational motion characteristics in the pitch and roll directions. Thus, similar to the analysis for ${}^B\mathbf{R}_A$, we can derive the rotation matrix ${}^A\mathbf{R}_D$ as follows:

$${}^A\mathbf{R}_D = \mathbf{R}_{\text{rot}(v_s)}(\phi) \mathbf{R}_{\text{rot}(u_s)}(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \sin \phi & \cos \phi \sin \phi \\ 0 & \cos \phi & -\sin \phi \\ -\sin \phi & \cos \phi \sin \phi & \cos \phi \cos \phi \end{bmatrix} \quad (2)$$

where ϕ and ϕ are the rotation angles of the built-in WEC about the u_s and v_s axes in the Spar's coordinate system $\{A\}$, respectively.

The multi-body dynamic model of the Spar-WECDP can be formulated by Newton-Euler method, which can be written as

$$\mathbf{M}_s \begin{bmatrix} \ddot{\mathbf{x}}_s \\ \ddot{\boldsymbol{\theta}}_s \end{bmatrix} + \mathbf{G}_s = \mathbf{F}_{wf} + \mathbf{F}_p + \mathbf{F}_{line} \quad (3)$$

$$\mathbf{M}_c \begin{bmatrix} \ddot{\mathbf{x}}_c \\ \ddot{\boldsymbol{\theta}}_c \end{bmatrix} + \mathbf{G}_c = -\mathbf{F}_p \quad (4)$$

where $\mathbf{M}_s$ and $\mathbf{M}_c$ are the inertia items, with subscripts s and c representing the Spar and the WECDP, respectively. $\mathbf{G}_s$ and $\mathbf{G}_c$ represent the respective gravity terms. $\ddot{\mathbf{x}}_s$ and $\ddot{\boldsymbol{\theta}}_s$ are the translation acceleration vector and rotation acceleration vector of Spar, respectively. $\ddot{\mathbf{x}}_c$ and $\ddot{\boldsymbol{\theta}}_c$ are the translation acceleration vector and rotation acceleration vector of WECDP, respectively. $\mathbf{F}_{wf}$ is the wave force acting on the Spar, $\mathbf{F}_p$ is the PTO force acting on the Spar, and $\mathbf{F}_{line}$ is the mooring force acting on Spar.

Due to the rotation of Spar and WECDP in wave excitation, its inertia items $\mathbf{M}_s$ and $\mathbf{M}_c$ can be expressed as

$$\mathbf{M}_s = \begin{bmatrix} m_s \mathbf{E}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & {}^B\mathbf{R}_A {}^A\mathbf{I}_s ({}^B\mathbf{R}_A)^T \end{bmatrix} \quad (5)$$

$$\mathbf{M}_c = \begin{bmatrix} m_c \mathbf{E}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & {}^B \mathbf{R}_D {}^D \mathbf{I}_c ({}^B \mathbf{R}_D)^T \end{bmatrix} \quad (6)$$

where the ${}^A \mathbf{I}_s$ is the moment of inertia matrix of the WECDP in the moving coordinate system $A - u_s v_s w_s$. The ${}^D \mathbf{I}_c$ is the moment of inertia matrix of the WECDP in the local coordinate system $F - u_c v_c w_c$. ${}^B \mathbf{R}_D = {}^B \mathbf{R}_A {}^A \mathbf{R}_D$ denotes the orientation of WECDP in the fixed coordinate system $B - xyz$.

In fixed coordinate system $B - xyz$, the gravity terms for Spar and WECDP can be expressed as

$$\mathbf{G}_a = [0 \ 0 \ m_a g \ 0 \ 0 \ 0]^T, (a = s, c) \quad (7)$$

3.2. Wave force and mooring force acting on the spar

The wave propagates in the ζ direction relative to the x -axis, and the wave force acting on the Spar is described as follows within the context of potential flow theory.

$$\mathbf{F}_{wfj} = \mathbf{F}_{excj} + \mathbf{F}_{radj} + \mathbf{F}_{resj}, (j = 1, 2, 3, 4, 5) \quad (8)$$

where j represents the motion mode of the Spar, with 1, 2, 3, 4, 5 correspond to surge, sway, heave, roll, and pitch, respectively. $\mathbf{F}_{excj}$ represents the wave excitation force or moment in the j -mode, $\mathbf{F}_{radj}$ denotes the wave radiation force or moment in the j -mode due to the Spar's motion, and $\mathbf{F}_{resj}$ is the hydrostatic restoring force or moment in the j -mode. Specifically.

- (1) For wave excitation force or moment, they can be expressed under different wave conditions as follows (Gao et al., 2024; Vicente et al., 2013).

$$\mathbf{F}_{excj} = \begin{cases} \frac{H}{2} \text{Re}\{F_{RAOj}(\omega_n)\} \cos(\omega_n t + \zeta_n + \theta_n) \dots \dots \dots \text{Regular wave} \\ \sum_{n=1}^N \sqrt{2S(\omega_n) \Delta \omega} \text{Re}\{F_{RAOj}(\omega_n)\} \cos(\omega_n t + \zeta_n + \theta_n) \dots \text{Irregular wave} \end{cases} \quad (9)$$

where H represents the regular wave height, $\text{Re}\{F_{RAOj}(\omega_n)\}$ is wave excitation force or moment modulus per wave amplitude in the j -mode for wave frequency ω_n , and θ_n represents the corresponding phase angle. An irregular wave is represented as the sum of multiple regular waves, with N denoting the number of regular wave components. The wave frequencies range from 0.100 rad/s and 3.141 rad/s, incremented by $\Delta \omega = 0.032$ rad/s. ζ_n and $S(\omega)$ are random phase angle and the spectral

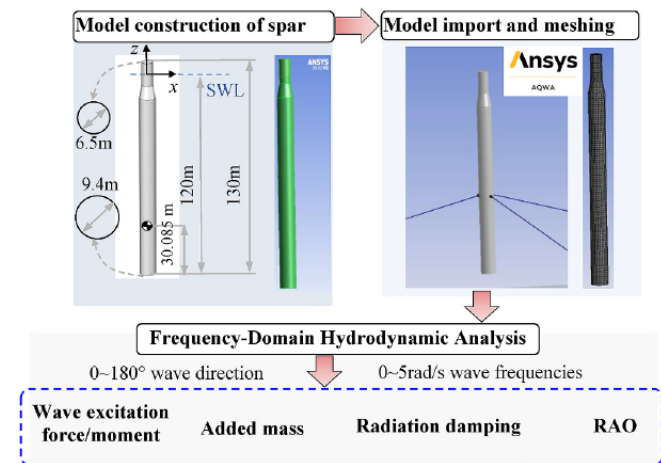


Fig. 12. Flowchart for calculating hydrodynamic parameters of the Spar using ANSYS AQWA.

density of the irregular wave, respectively. As a result, the wave excitation force acting on Spar can be expressed as

$$\mathbf{F}_{exc} = [F_{exc,1} \ F_{exc,2} \ F_{exc,3} \ F_{exc,4} \ F_{exc,5} \ 0]^T \quad (10)$$

The hydrodynamic parameters, including the modulus and phase of wave excitation force or moment, can be obtained by meshing the Spar model and solving it numerically in ANSYS AQWA (Zou and Abdelkhalik, 2020; Wei et al., 2024). As shown in Fig. 12, the Spar geometry model is first established based on its dimensions, followed by importing the model and generating the mesh. The still waterline, wave field conditions, and mooring line constraints are then set within the model. The incident wave angle can be chosen from 0° to 180° with an interval of 22.5° . The wave frequency ranges from 0 rad/s to 5 rad/s during the calculation. Finally, the ANSYS AQWA solver is used to calculate the wave excitation force/moment $\mathbf{F}_{RAO}$. Note that other hydrodynamic parameters, including added mass, radiation damping, and RAO, can also be obtained using this calculation process.

Fig. 13 presents this calculation results for the surge, heave, and pitch modes and validates them through published numerical analyses. It can be observed that the results obtained from ANSYS AQWA align well with the published numerical results (Jonkman, 2010) from NREL. Notably, since the Spar is axisymmetric, the hydrodynamic parameters for surge and sway are identical, as are those for pitch and roll. Therefore, the hydrodynamic parameters for the sway and roll directions are not repeated in this paper.

- (2) According to Cummins equation (Cummins, 1962), the radiation force $\mathbf{F}_{rad}$ can be expressed by convolution term

$$\mathbf{F}_{rad} = -\mathbf{A}(\infty) \begin{bmatrix} \dot{\mathbf{x}}_s(t) \\ \dot{\boldsymbol{\theta}}_s(t) \end{bmatrix} - \int_0^t \mathbf{K}(t-\tau) \begin{bmatrix} \dot{\mathbf{x}}_s(\tau) \\ \dot{\boldsymbol{\theta}}_s(\tau) \end{bmatrix} d\tau \quad (11)$$

where $\mathbf{A}(\infty)$ is the infinite-frequency added mass, $\mathbf{K}(t)$ represents the retardation function that characterizes the radiation damping force. These two terms are the critical to represent the radiation force.

For the added mass, it can be determined using frequency-domain identification in MATLAB, where the input is frequency and the output is added mass. Fig. 14 present this identification results and compare it with the published numerical results (Jonkman, 2010) from NREL.

The retardation function $\mathbf{K}(t)$ represents a linear system, with its solution typically approximated through the state-space model substitution approach (Taghipour et al., 2008). For brevity, the detailed solution process is not provided in this paper.

$$\begin{cases} \int_0^t \mathbf{K}(t-\tau) \begin{bmatrix} \dot{\mathbf{x}}_s(\tau) \\ \dot{\boldsymbol{\theta}}_s(\tau) \end{bmatrix} d\tau = \mathbf{C} \boldsymbol{\mu}(t) + \mathbf{D} \begin{bmatrix} \dot{\mathbf{x}}_s(t) \\ \dot{\boldsymbol{\theta}}_s(t) \end{bmatrix} \\ \dot{\boldsymbol{\mu}}(t) = \mathbf{A}' \boldsymbol{\mu}(t) + \mathbf{B}' \begin{bmatrix} \dot{\mathbf{x}}_s(t) \\ \dot{\boldsymbol{\theta}}_s(t) \end{bmatrix} \end{cases} \quad (12)$$

where the $\boldsymbol{\mu}(t)$ represents the state variables of the state space model. The $\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}$ and $\mathbf{D}$ represent constant coefficients matrices that characterize the Spar's radiation damping force. Fig. 15 presents a comparison between the retardation function $\mathbf{K}(t)$ derived from the state-space model and the published numerical results (Jonkman, 2010) from NREL. The results show strong agreement, demonstrating that the state-space model effectively represents the characteristics of the retardation function.

- (3) The hydrostatic restoring force is zero in surge and sway direction. due to the influence of pitch and roll on the vertical restoring force (John Nicholas, 1979), the hydrostatic restoring force in the heave direction is expressed as

$$\mathbf{F}_{res,3} = \rho g \Delta V_d - \rho g \alpha y_f A_w + \rho g \beta x_f A_w \quad (13)$$

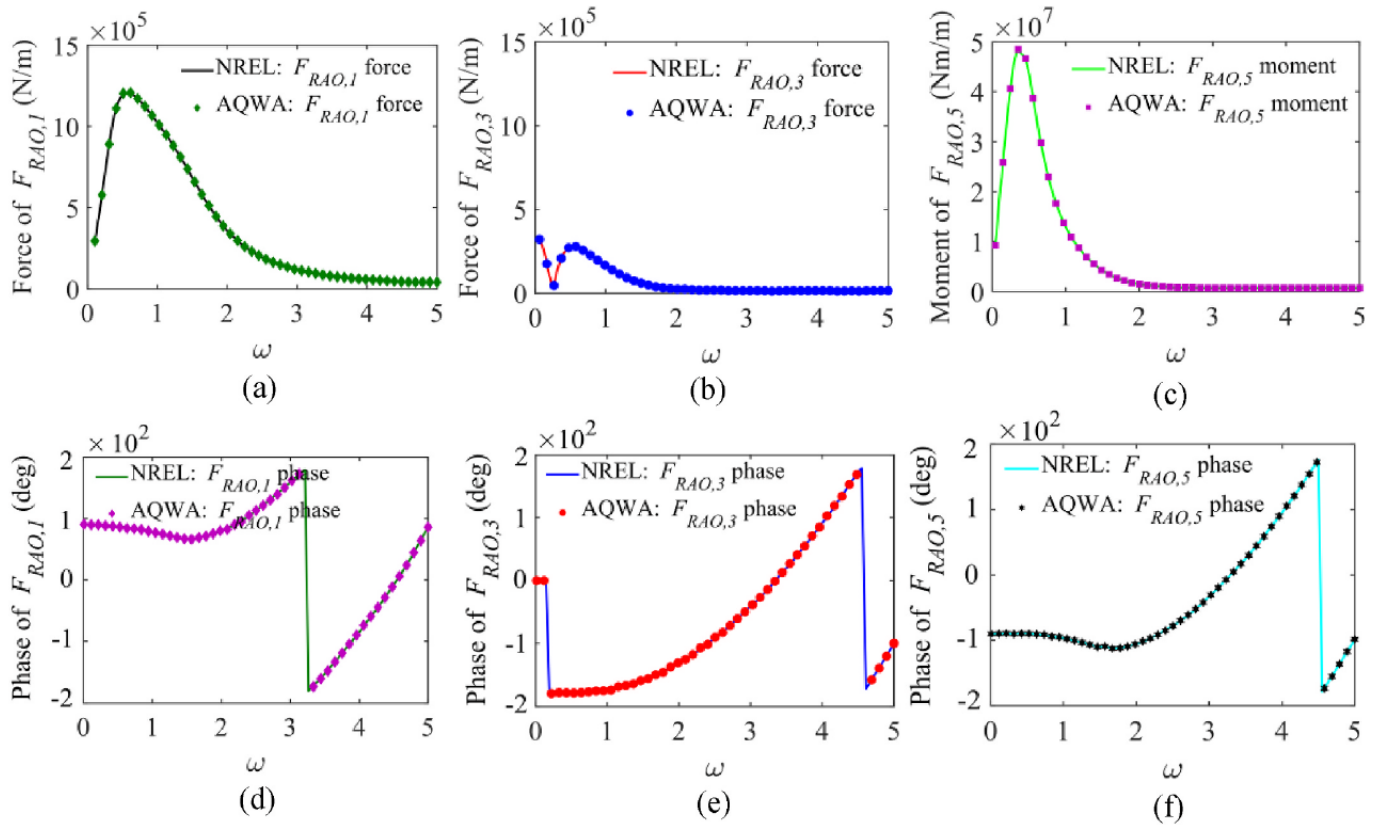


Fig. 13. The wave excitation force/moment comparison for the Spar between the National Renewable Energy Laboratory and ANSYS AQWA: (a) magnitude of $F_{RAO,1}$, (b) magnitude of $F_{RAO,3}$, (c) magnitude of $F_{RAO,5}$, (d) phase of $F_{RAO,1}$, (e) phase of $F_{RAO,3}$ and (f) phase of $F_{RAO,5}$.

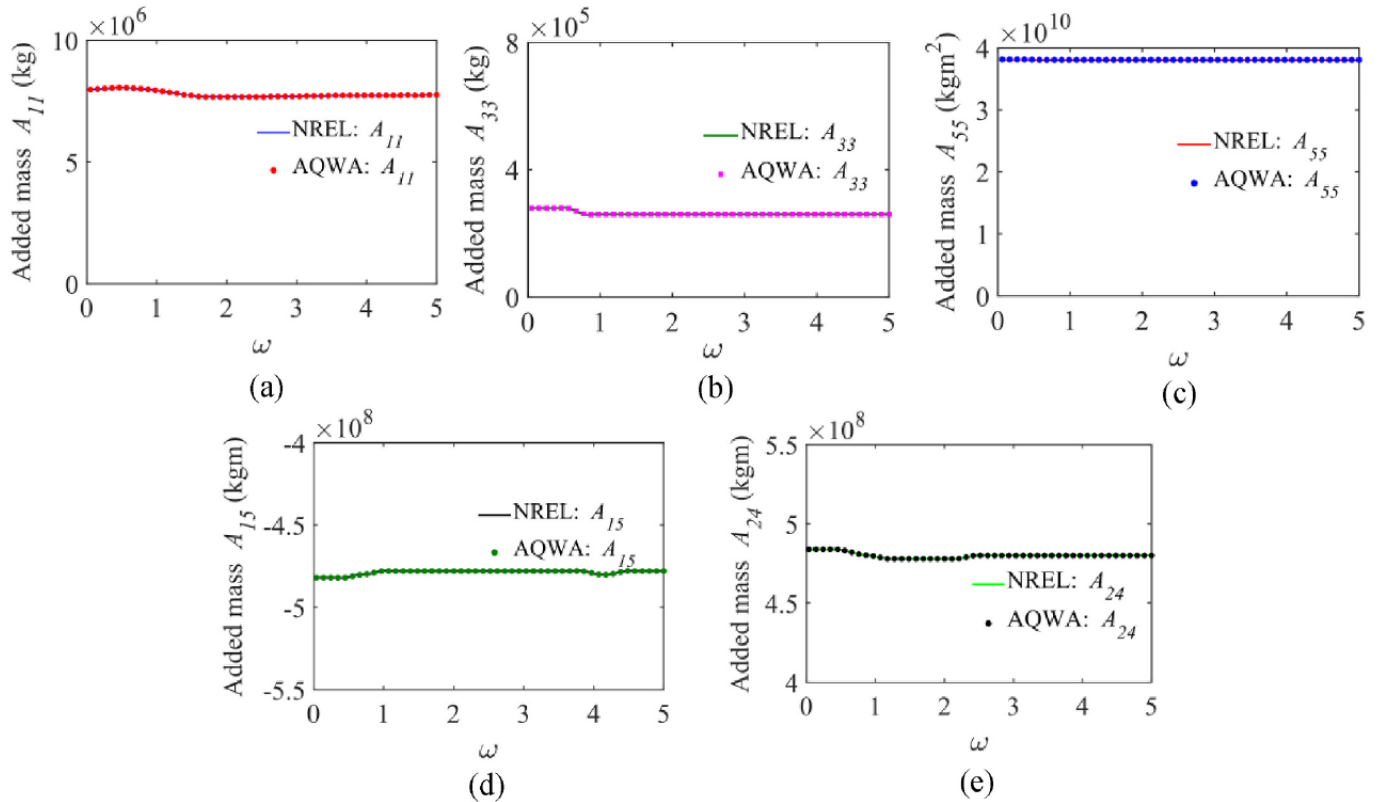


Fig. 14. The added mass comparison for the Spar between the National Renewable Energy Laboratory and ANSYS AQWA: (a) magnitude of $F_{RAO,1}$, (b) magnitude of $F_{RAO,3}$, (c) magnitude of $F_{RAO,5}$, (d) phase of $F_{RAO,1}$, (e) phase of $F_{RAO,3}$ and (f) phase of $F_{RAO,5}$.

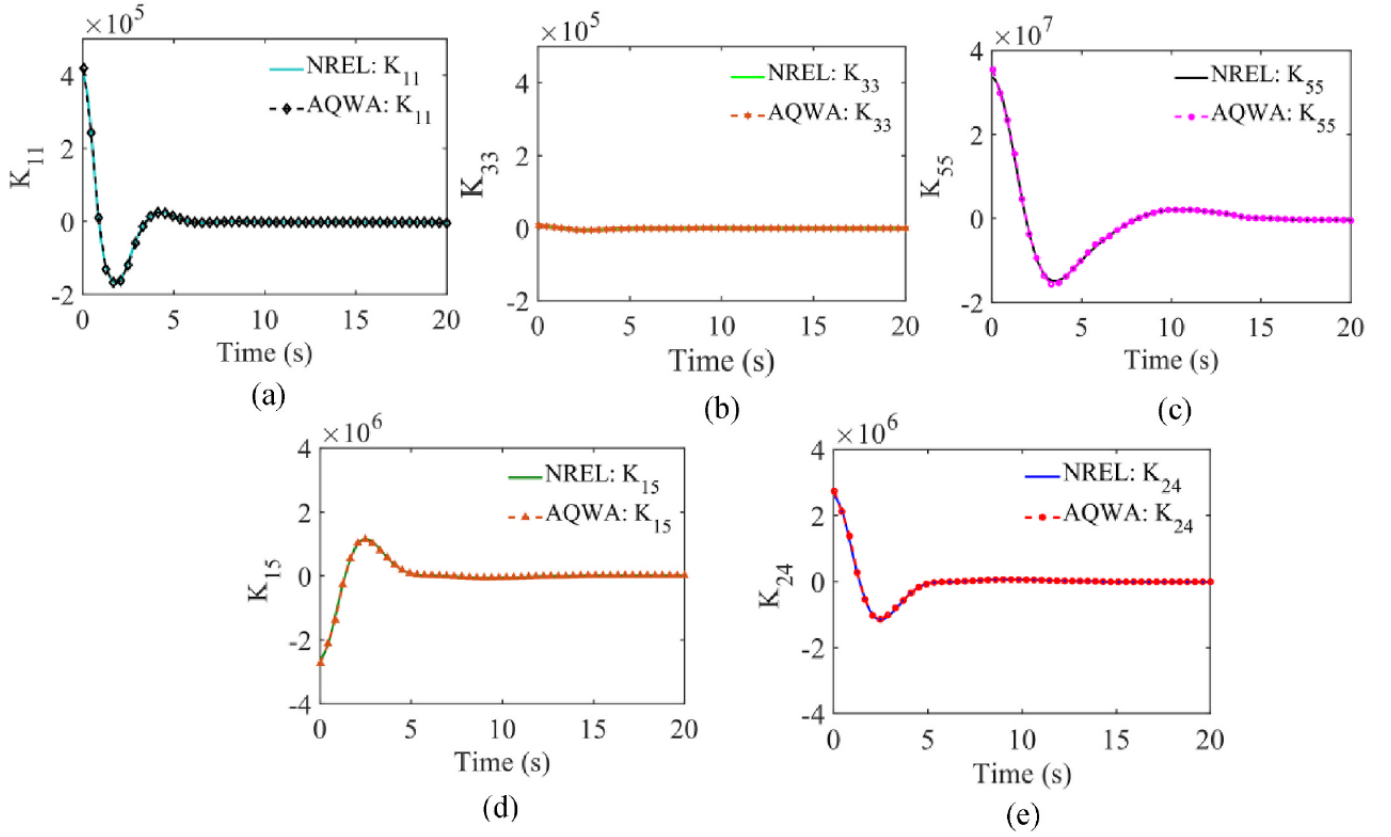


Fig. 15. The comparison of retardation function for the Spar between the National Renewable Energy Laboratory (NREL) and ANSYS AQWA: (a) K_{11} , (b) K_{33} , (c) K_{55} , (d) K_{15} and (e) K_{24} .

where g and ρ represent the acceleration of gravity and sea-water density, respectively. ΔV_d is the displaced volume change of the seawater. x_f and y_f represent the coordinates of the center of flotation. α , β and A_w represent roll angle, pitch angle and waterplane area of Spar, respectively.

In the roll and pitch directions, the hydrostatic restoring moment (Gao et al., 2024) is expressed as

$$F_{res,4} = -\rho g z y_f A_w - k_{44} \alpha \quad (14)$$

$$F_{res,5} = \rho g z x_f A_w - k_{55} \beta \quad (15)$$

where k_{44} and k_{55} are the hydrostatic stiffness in roll and pitch, respectively. z is the Spar displacement in heave direction. As a result, the hydrostatic restoring force and moment acting on Spar can be expressed as

$$\mathbf{F}_{res} = [0 \quad 0 \quad F_{res,3} \quad F_{res,4} \quad F_{res,5}]^T \quad (16)$$

(4) Based on the disclosed structural data of the Spar floating platform from NREL, reasonable assumptions are made to simplify

the analysis of the mooring system, including linear compliance of the three mooring lines and the neglect of mooring inertia and damping (Jonkman, 2010). Meanwhile, these assumptions are also intended to better align with the hydrodynamic analysis results published by NREL in reference (Jonkman, 2010; Ramachandran et al.). Consequently, the mooring force on the Spar can be expressed as follows:

$$\mathbf{F}_{line} = \mathbf{F}_{wave,0} - \mathbf{K}_{line} \begin{bmatrix} \mathbf{x}_s \\ \boldsymbol{\theta}_s \end{bmatrix} \quad (17)$$

where $\mathbf{F}_{wave,0}$ is the total force exerted on the Spar by the entire mooring lines when the Spar is at rest, and $\mathbf{K}_{line}$ is the stiffness matrix of the mooring lines. These parameters can be calculated by performing a linearization analysis in the FAST code. This linearization analysis involves independently perturbing the Spar's multi-DOF motion and measuring the resulting variations in the mooring forces. Reference (Jonkman, 2010) presents the calculation results. Specifically,

$$\mathbf{F}_{wave,0} = [0 \quad 0 \quad -1.607 \times 10^6 \text{ N} \quad 0 \quad 0]^T \quad (18)$$

$$\mathbf{K}_{line} = \begin{bmatrix} 4.118 \times 10^4 \text{ N/m} & 0 & 0 & 0 & -2.821 \times 10^6 \text{ N/rad} \\ 0 & 4.118 \times 10^4 \text{ N/m} & 0 & 2.821 \times 10^6 \text{ N/rad} & 0 \\ 0 & 0 & 1.194 \times 10^4 \text{ N/m} & 0 & 0 \\ -2.816 \times 10^6 \text{ Nm/m} & 0 & 0 & 3.111 \times 10^8 \text{ Nm/rad} & 0 \\ 0 & 0 & 0 & 3.111 \times 10^8 \text{ Nm/rad} & 0 \end{bmatrix} \quad (19)$$

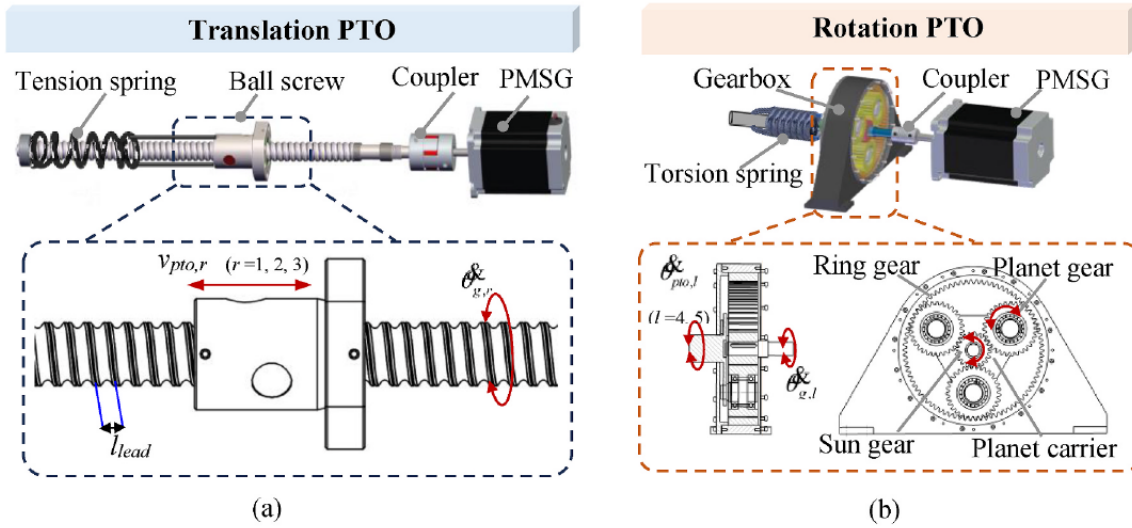


Fig. 16. Schematic of PTO: (a) Translation PTO, (b) rotation PTO.

Note that, the Spar's yaw motion is nearly zero due to the symmetry of the spar, and is therefore neglected in this study. As a result, the stiffness matrix $\mathbf{K}_{line}$ is 5×5 , and the force vector $\mathbf{F}_{wave,0}$ is 5×1 .

3.3. PTO force acting on the spar

The PTO is an energy conversion device, and its force is directly

As shown in Fig. 16(a), in the operation process of the translation PTO, the ball screw, coupler, and PMSG rotor only contain rotation. The tension spring provides stiffness for the translation PTO. Meanwhile, since above components are attached to the Spar platform, their mass is directly included in the Spar's mass for simplification. Thus, in the moving coordinate system $A - u_s v_s w_s$, the forces of translation PTOs acting on the Spar can be obtained.

$${}^A \mathbf{F}_{pto,r} = \left\{ \frac{2\pi}{\lambda l_{lead}} \left[\tau_{PMSG,r} + (I_{S,r} + I_{C,r} + I_{P,r}) \frac{2\pi}{l_{lead}} \ddot{x}_{pto,r} \right] + k_{T,r} x_{pto,r} \right\} \begin{bmatrix} \delta_{1r} \\ \delta_{2r} \\ \delta_{3r} \end{bmatrix}, (r=1, 2, 3) \quad (23)$$

related to its input velocity (Chen, 2024; Yang et al., 2024). Specifically.

- (1) For translation PTO, the input linear velocity equals the velocity difference between the WECDP and the Spar along the PTO's operation direction. However, due to the rotation of the Spar, the linear velocities of the Spar and the WECDP in a fixed coordinate system are not aligned with the translation PTO's operation direction. To eliminate this effect, we convert the linear velocities of the Spar and WECDP into the Spar's moving coordinate system $A - u_s v_s w_s$.

$${}^A \dot{\mathbf{x}}_a = {}^A \mathbf{R}_B \dot{\mathbf{x}}_a, (a=s, c) \quad (20)$$

Moreover, since the rotation center F of the WECDP does not coincide with its mass center D , the difference between ${}^A \dot{\mathbf{x}}_s$ and ${}^A \dot{\mathbf{x}}_c$ does not represent the PTO input velocity. Based on the distance l_{DF} between points D and F , the linear velocity of the rotation center F can be obtained as follows:

$${}^A \dot{\mathbf{x}}_F = {}^A \dot{\mathbf{x}}_c + (\dot{\theta}_s - \dot{\theta}_c) \times [0 \ 0 \ l_{DF}]^T \quad (21)$$

Thus, according to Eqs. (20) and (21), the input linear velocity of translation PTO are given as follows:

$$[\dot{x}_{pto,1} \ \dot{x}_{pto,2} \ \dot{x}_{pto,3}]^T = {}^A \mathbf{R}_B \dot{\mathbf{x}}_s - {}^A \mathbf{R}_B \dot{\mathbf{x}}_c - (\dot{\theta}_s - \dot{\theta}_c) \times [0 \ 0 \ l_{DF}]^T \quad (22)$$

where the subscripts 1, 2, and 3 represent the translation PTO in the surge, sway, and heave directions, respectively.

where $I_{S,r}$, $I_{C,r}$, $I_{P,r}$ and $\tau_{PMSG,r}$ represent the rotational inertia of the ball screw, coupler, and PMSG rotor and PMSG's electromagnetic moment for the corresponding translation PTO, respectively. $k_{T,r}$ is the stiffness of tension spring. λ denotes the transmission efficiency of the translation PTO accounting for its operational friction. l_{lead} and δ denote the lead of the ball screw and Kronecker delta function, respectively.

Describe Eq. (23) in the fixed coordinate system $B - xyz$.

$${}^B \mathbf{F}_{pto,r} = {}^B \mathbf{R}_A {}^A \mathbf{F}_{pto,r} \quad (24)$$

- (2) For rotation PTO, its input angle velocity can be calculated according to the relative motion angle of Spar and WECDP.

$$[\dot{\theta}_{pto,4} \ \dot{\theta}_{pto,5} \ 0]^T = \dot{\theta}_s - \dot{\theta}_c \quad (25)$$

where the subscripts 4, and 5 represent the rotation PTO in the roll and pitch directions, respectively.

As shown in Fig. 16(b), the rotation motion of the gearbox, coupler, and PMSG is involved in the rotation PTO's power generation process. The torsion spring provides stiffness for the rotation PTO. Similar to the translation PTO, the forces of rotation PTOs acting on the Spar can be expressed as follows:

$${}^A\mathbf{F}_{pto,l} = \left\{ \frac{1+\alpha}{\eta} \left[\tau_{G,l} + (I_{G,l} + I_{C,l} + I_{P,l}) \left(\ddot{\theta}_{pto,l} + \alpha \ddot{\theta}_{pto,l} \right) \right] + k_{s,l} \theta_{pto,l} \right\} \begin{bmatrix} \delta_{4l} \\ \delta_{5l} \\ 0 \end{bmatrix}, (l=4,5) \quad (26)$$

where $I_{G,l}$, $I_{C,l}$, $I_{P,l}$ and $\tau_{PMSG,l}$ represent the rotational inertia of the gearbox, coupler, and PMSG rotor and PMSG's electromagnetic moment in the corresponding rotation PTO, respectively. $k_{s,l}$ is the stiffness of torsion spring, η denotes the transmission efficiency of the rotation PTO accounting for its operational friction, and α is the transmission ratio of gearbox.

By describing Eq. (26) in the fixed coordinate system B -xyz and combining with Eq. (24), we can obtain the PTO force acting on the Spar.

$$\mathbf{F}_p = \begin{bmatrix} {}^B R_A \sum_{r=1}^3 {}^A \mathbf{F}_{pto,r} \\ {}^B R_A \sum_{l=4}^5 {}^A \mathbf{F}_{pto,l} \end{bmatrix} \quad (27)$$

3.4. Vibration response and energy harvesting of the Spar-WECDP

To obtain the vibration response of the Spar-WECDP, by substituting Eqs. (5)–(8), (10)–(12), (16), (17) and (27) into Eqs. (3) and (4), we can obtain

$$\begin{cases} [\mathbf{M}_s + \mathbf{A}(\infty)] \begin{bmatrix} \ddot{\mathbf{x}}_s(t) \\ \ddot{\theta}_s(t) \end{bmatrix} + \mathbf{D} \begin{bmatrix} \dot{\mathbf{x}}_s(t) \\ \dot{\theta}_s(t) \end{bmatrix} + \mathbf{C} \boldsymbol{\mu}(t) = \mathbf{F}_{exc} + \mathbf{F}_{res} + \begin{bmatrix} {}^B R_A \sum_{r=1}^3 {}^A \mathbf{F}_{pto,r} \\ {}^B R_A \sum_{l=4}^5 {}^A \mathbf{F}_{pto,l} \end{bmatrix} + \mathbf{F}_{wave,0} - \mathbf{K}_{line} \begin{bmatrix} \mathbf{x}_s \\ \theta_s \end{bmatrix} - \begin{bmatrix} \mathbf{0}_{1 \times 2} \\ m_s \mathbf{g} \\ \mathbf{0}_{1 \times 3} \end{bmatrix} \\ \mathbf{M}_c \begin{bmatrix} \ddot{\mathbf{x}}_c(t) \\ \ddot{\theta}_c(t) \end{bmatrix} = - \begin{bmatrix} {}^B R_A \sum_{r=1}^3 {}^A \mathbf{F}_{pto,r} \\ {}^B R_A \sum_{l=4}^5 {}^A \mathbf{F}_{pto,l} \end{bmatrix} - \begin{bmatrix} \mathbf{0}_{1 \times 2} \\ m_c \mathbf{g} \\ \mathbf{0}_{1 \times 3} \end{bmatrix} \\ \dot{\boldsymbol{\mu}}(t) = \mathbf{A}' \boldsymbol{\mu}(t) + \mathbf{B}' \begin{bmatrix} \dot{\mathbf{x}}_s(t) \\ \dot{\theta}_s(t) \end{bmatrix} \end{cases} \quad (28)$$

The above differential equation set (28) can be solved by Runge-Kutta approach (Blum, 1962), thus obtaining the motion of both the Spar and WECDP across various directions.

In the energy harvesting process, since the PMSG generates electricity as alternating current (AC), the generated AC must be converted into direct current (DC) through an AC/DC rectifier when storing electric energy. Then, it is stabilized by a DC/DC converter to charge the battery. To obtain the output power of the WECDP, we present the equivalent circuit of the energy harvesting structure in Fig. 17 based on circuit theory.

As shown in Fig. 17, rectifier (AC/DC) and power controller (DC/DC) are achieved using three-phase full-bridge passive circuits and three-phase interleaved parallel circuits, respectively. On the battery side, an open-circuit voltage source V_o in series with an internal resistance R_{int} models the battery's operating mechanism. When integrating circuit dynamics, the resistances of inductors, capacitors, and switching

elements are generally small and are ignored in this modeling process.

According to the circuit impedance matching principle (Lassioui et al., 2020; Wu et al., 2011), the power converter and the battery can be equivalent to the load resistance of the PMSG. Thus, based on the PMSG equivalent model extensively studied in the literature (Zhang et al., 2016, 2024b; Jr and Lima, 2016; Sanchez, 2012), the electromagnetic moment can be expressed as follows.

$$\tau_{PMSG,q} = \frac{3k_e^2 \omega_q}{6R_a + 4R_{L,q}}, (q=r,l) \quad (29)$$

where $R_{L,q}$ and ω_q denote equivalent load resistance and mechanical angular velocity of the PMSG in the corresponding PTO, respectively. R_a and k_e represent stator phase resistance, and voltage constant of PMSG, respectively.

Furthermore, the rectifier (AC/DC) consists of passive circuits and has no impact on the equivalent load resistance $R_{L,q}$ of the PMSG. While the DC/DC converter includes switching element (MOSFET), which can adjust the equivalent load resistance $R_{L,q}$ by changing the duty cycle of MOSFET (Xu et al., 2021; Dong et al., 2023). According to the circuit impedance matching principle, the relationship between the equivalent

load resistance and duty cycle can be expressed as follows:

$$R_{L,q} = \left(\frac{1-D_q}{D_q} \right)^2 R_{int}, (q=r,l) \quad (30)$$

where D_q denote the duty cycle of the DC/DC converter in the corresponding PTO. Combining Eqs. (29) and (30), the equivalent electromagnetic damping of each PMSG can be obtained as:

$$c_{PMSG,q} = \frac{3k_e^2}{6R_a + 4 \left(\frac{1-D_q}{D_q} \right)^2 R_{int}}, (q=r,l) \quad (31)$$

The WECDP includes two types of PTOs: translation and rotation. Based on their respective mechanical transmission structures, the mechanical angular velocity ω_q can be expressed as:

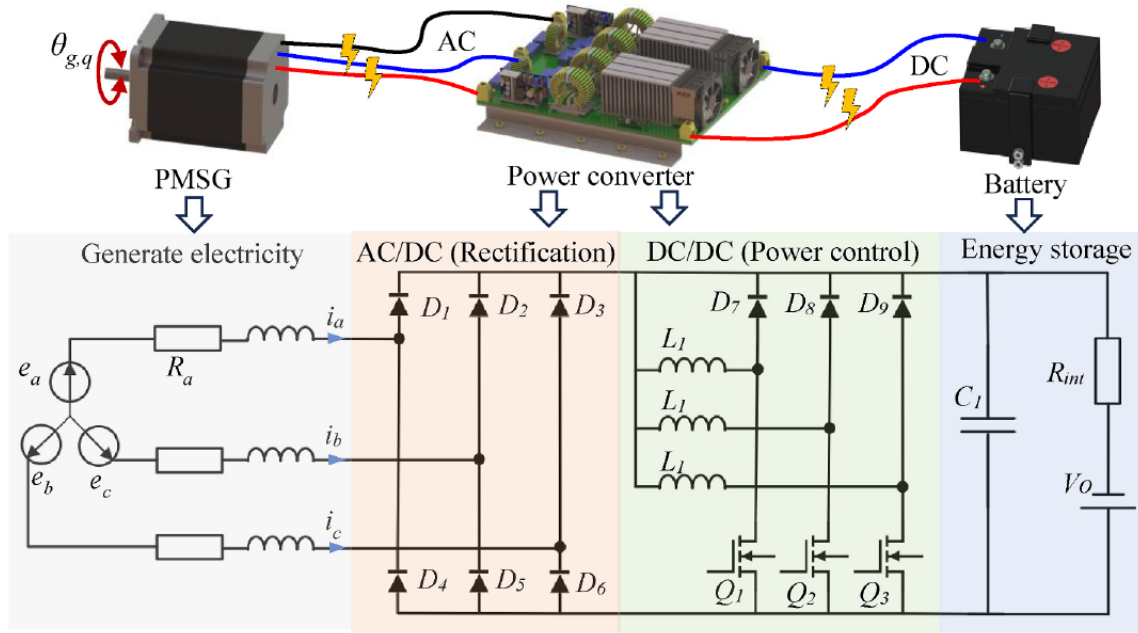


Fig. 17. Schematic of energy harvesting.

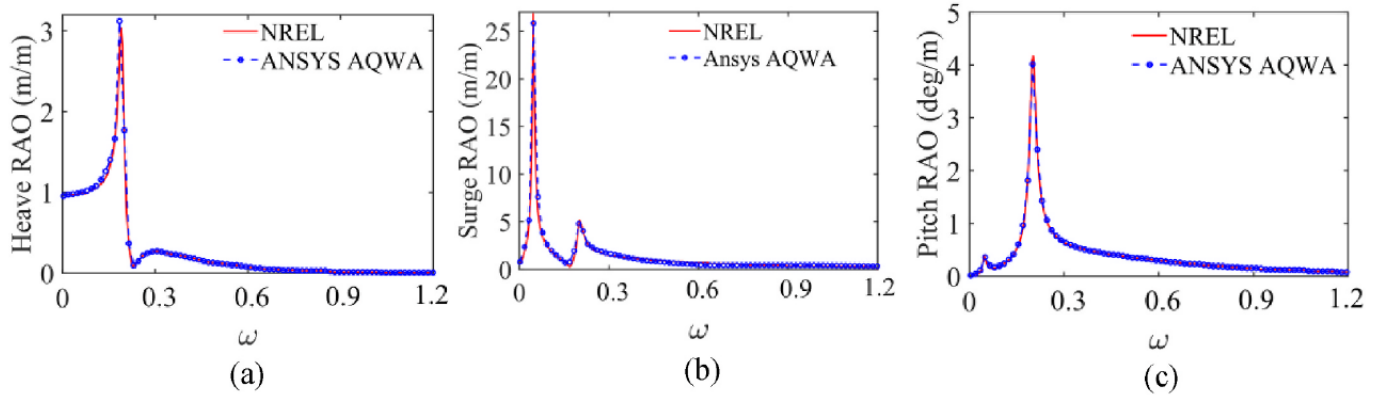


Fig. 18. Comparison of Spar RAO results between the National Renewable Energy Laboratory and ANSYS AQWA: (a) Heave RAO (b) Surge RAO and (c) Pitch RAO.

$$\omega_q = \begin{cases} \frac{2\pi}{l_{lead}} \dot{v}_{pto,r}, (q = r) \\ \dot{\theta}_{pto,l} + \alpha \dot{\theta}_{pto,l}, (q = l) \end{cases} \quad (32)$$

As a result, combining Eqs. (31) and (32), the overall output power P_{all} of Spar-WECDP can be calculated as follows:

$$P_{all} = \sum_{r=1}^3 c_{PMSG,r} \left(\frac{2\pi}{l_{lead}} \dot{v}_{pto,r} \right)^2 + \sum_{l=4}^5 c_{PMSG,l} (\dot{\theta}_{pto,l} + \alpha \dot{\theta}_{pto,l})^2 \quad (33)$$

The mean output power of Spar-WECDP can be expressed as:

$$\bar{P} = \frac{\int_{t_0}^{t_f} P_{all} dt}{t_f - t_0} \quad (34)$$

where t_0 and t_f denote the start and end times of the PMSG operation, respectively.

4. Simulation for validating the mathematical model

To validate the theoretical model, this section compares the simulation results of the Spar-WECDP under irregular waves with those from

numerical analysis.

4.1. Development of the simulation model

In our proposed system, the WEC with multiple DOFs is built into the Spar floating platform. This results in a multi-physical domain model that integrates the Spar's hydrodynamics, the mechanical dynamics between the Spar and WEC, and the electrodynamics of the WEC. Regarding the Spar's hydrodynamics, the ANSYS AQWA is used to calculate the motion response of the Spar. As shown in Fig. 18, the response amplitude operator (RAO) obtained from ANSYS AQWA aligns well with the published numerical results (Ramachandran et al.) from NREL. This consistency validates the hydrodynamic results presented in this study and lays the foundation for subsequent analysis.

Regarding mechanical dynamics and electrodynamics, the integration of ANSYS-AQWA and Simulink (Albert et al., 2017) is used to validate the theoretical model's accuracy. This method allows effective coupling of modules defining hydrostatic and hydrodynamic forces, and mechanical-electrical conversion (Albert et al., 2017; Bruzzzone et al., 2020). Fig. 19 presents the detailed simulation model framework.

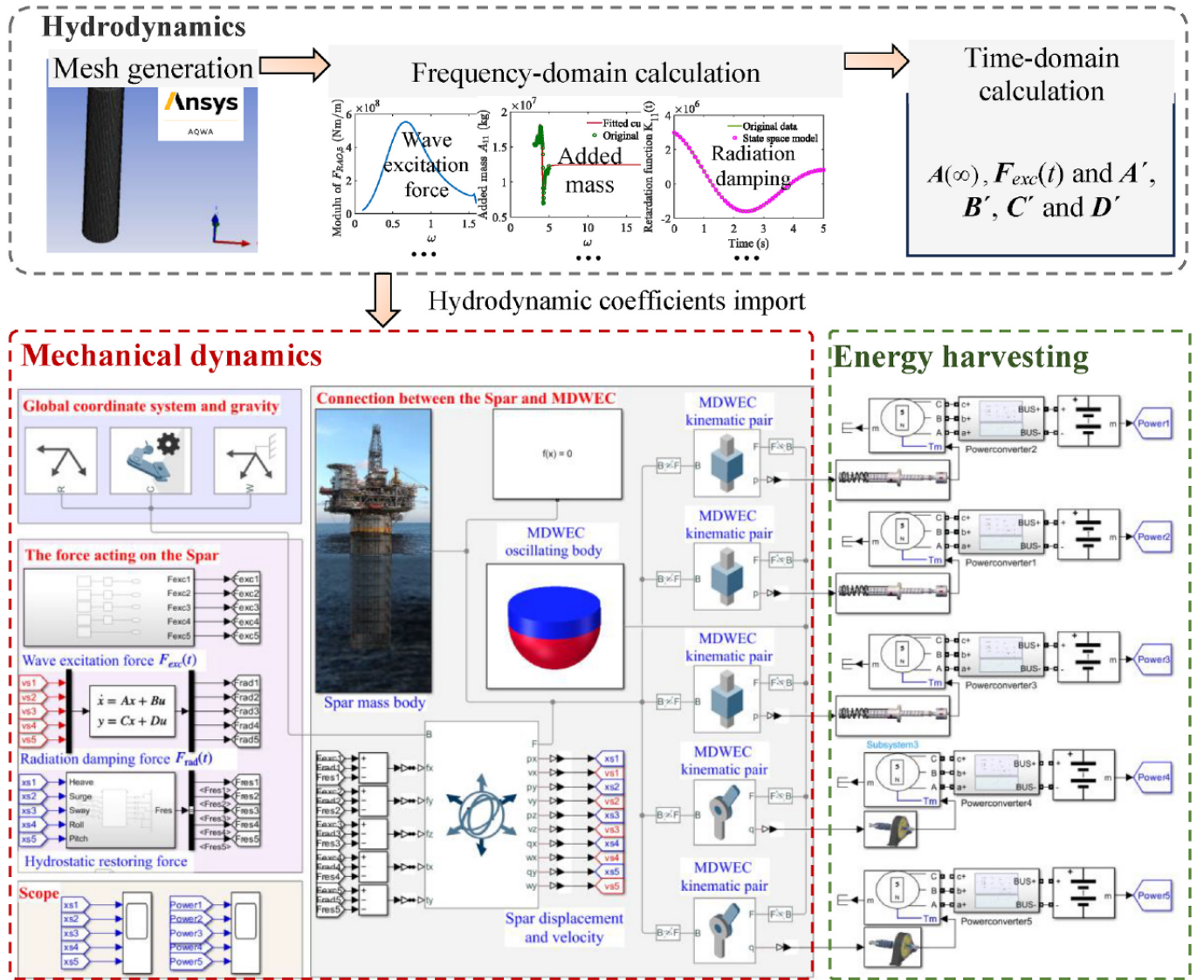


Fig. 19. Framework of the simulation model for Spar-WECDP.

- (1) **For hydrodynamics:** Initially, ANSYS-AQWA is utilized to generate the Spar mesh and compute hydrodynamic coefficients, such as wave excitation force, mooring force, added mass, and radiation damping within the frequency domain. Based on this, MATLAB is employed to compute the time domain wave excitation force F_{exc} , hydrostatic restoring force and moment F_{res} , the constant coefficient matrices and the infinite-frequency added mass $A(\infty)$.
- (2) **For mechanical dynamics:** Define the fixed coordinate system and apply gravitational forces. Next, link the mass bodies of the

Spar and WECDP through kinematic pairs. Load the Spar's infinite-frequency added mass, along with the PTO stiffness and damping coefficients in the WECDP, into the respective mass bodies and kinematic pairs. Utilizing the pre-calculated hydrodynamic coefficients, develop modules for the wave forces acting on the Spar in the time domain.

- (3) **For energy harvesting:** According to the schematic of the energy harvesting structure, construct the power converter and battery model using the power electronics simulation module in Simulink and import the corresponding structural parameters. Combining the model of hydrodynamics and mechanics, simulate the motion of the Spar and the output power of the WECDP. Utilize the scope module to monitor and record the relevant data.

Table 3

Main structure parameters of the PTO.

Symbol	Parameter	Value	Unit
I_s	Rotational inertia of ball screw	2.14×10^3	kg·m ²
I_p	Rotational inertia of PMSG rotor	1.01×10^3	kg·m ²
I_c	Rotational inertia of coupler	0.35×10^3	kg·m ²
I_G	Rotational inertia of gearbox	1.36×10^3	kg·m ²
R_a	Stator phase resistance	2.62	Ohm
k_e	Voltage constant	9.819	Vpeak L-L/(rad/s)
L	Inductance	1.87	mH
R_{int}	Internal resistance of battery	5.5	Ohm
l_{lead}	Lead of ball screw	0.01	m

4.2. Model verification

In order to assess the model's accuracy, an irregular wave condition is applied, with characteristics defined by $H_{1/3} = 2$ m, $\omega_p = 0.6$ rad/s,

$\zeta = 150$ deg. The corresponding wave spectrum density function $S(\omega) =$

$16.43 \frac{\omega_p^4 H_{1/3}^2}{\pi^4 \omega^5} \exp\left(-\frac{\omega_p^4 65.74}{\pi^4 \omega^4}\right)$ is provided by the China National Marine

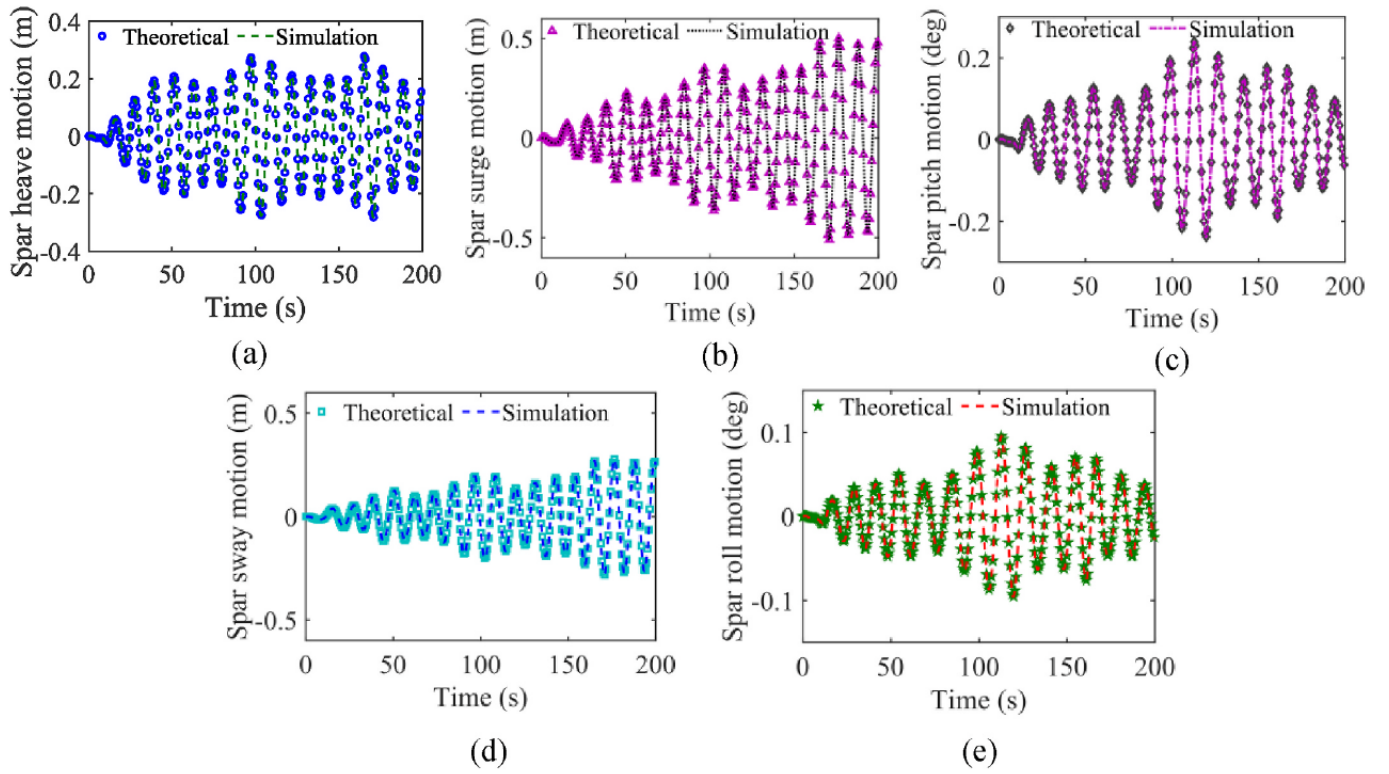


Fig. 20. The motion displacement validation of theoretical model under irregular wave condition: (a) Spar heave motion, (b) Spar surge motion, (c) Spar pitch motion, (d) Spar sway motion and (e) Spar roll motion.

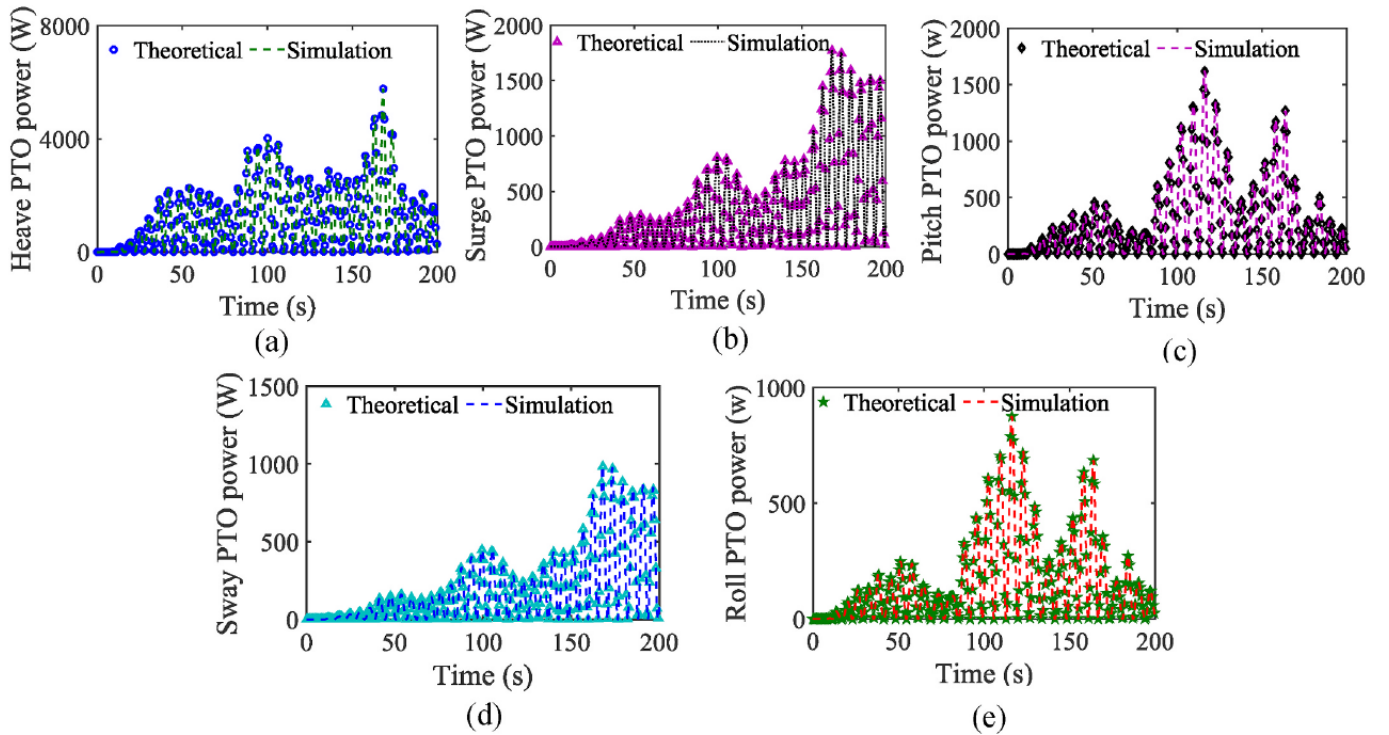


Fig. 21. The output power validation of theoretical model under irregular wave condition: (a) heave PTO power, (b) surge PTO power, (c) pitch PTO power, (d) sway PTO power and (e) roll PTO power.

Data Center (National Science & Technology Infrastructure), covering the northern South China Sea coastline. The PTO structural parameters used for the simulation analysis are provided in Table 3, and the other parameters for the simulation analysis include $k_{T,1} = 15 \text{ kN/m}$, $k_{T,2} = 15$

kN/m , $k_{T,3} = 10 \text{ kN/m}$, $k_{S,4} = 200 \text{ kN/m}$, $k_{S,5} = 200 \text{ kN/m}$, $c_{PM,SG,1} = 5 \text{ kNs/m}$, $c_{PM,SG,2} = 5 \text{ kNs/m}$, $c_{PM,SG,3} = 2 \text{ kNs/m}$, $c_{PM,SG,4} = 15 \text{ kNs/m}$, and $c_{PM,SG,5} = 15 \text{ kNs/m}$. Combining with above structural parameters, the vibration response and energy harvesting characteristics of the

Spar-WECDP are obtained in Simulink (See Fig. 20).

Figs. 20 and 21 illustrates the Spar's motion and the WECDP's output power under the given parameters, with the results from the numerical analysis (detailed in Section 3) closely matching those obtained from the model simulation. This consistency confirms the accuracy of the developed theoretical model and the computational approach employed.

5. Optimization framework and analysis

Based on the theoretical model, this section formulates the parameter optimization aimed at reducing the Spar's vibration amplitude in multiple motion directions. The details of the performance analysis are also presented.

5.1. Optimization framework

This optimization framework contains two primary elements: optimization objective and optimization method.

5.1.1. Optimization objective

Before determine the optimization objective, the response amplitude operator (RAO) is defined as the ratio of the Spar's vibration amplitude to the ocean wave amplitude, thus enabling a universal analysis for optimization results. Utilizing this non-dimensional RAO parameter can easily obtain the results under different wave heights.

The Spar experiences five-dimensional vibrations: surge, sway, heave, roll, and pitch under wave excitation. To ensure the Spar's operational stability, the optimization objective is defined as minimizing the sum of the maximum response amplitude operator (RAO) across all motion directions. Given that the RAO units for translational and rotational motions differ, the RAO of the Spar with the WEC is normalized by dividing it by the peak RAO of the Spar without the WEC in the same direction. This normalization results in a dimensionless value of 1, enabling the effective combination of rotational and translational movements.

Noted that, the resonant frequencies of the Spar are not the same across different DOFs. To ensure the optimization results are robust under practical wave conditions, the optimization should not focus on a single resonant frequency but instead consider a range of frequencies, covering all dangerous frequencies (i.e., resonant frequencies) that the Spar may encounter. In addition, if optimization is performed for each resonant frequency under different DOFs, the optimal PTO coefficients will inevitably differ for each resonant frequency. This means that when the wave frequency shifts from one resonant point to another, the PTO stiffness and damping must be adjusted to achieve effective vibration control. Thus, it is desirable for the optimal PTO coefficients to have good robustness, allowing them to effectively adapt to changes in wave frequency.

Following the fundamental principles of vibration control, the optimization is achieved by adjusting the stiffness $k_{T,q}$ and electromagnetic damping $c_{T,q}$ of each PTO in the WEC. The adjustment ranges of these parameters are incorporated into the optimization process to ensure practical feasibility. As a result, the optimization objective can be formulated as follows:

$$J = \min_{k_{T,q}, c_{T,q}} \sum_{j=1}^5 \left(\max_{\omega_1 \leq \omega \leq \omega_2} \left[\frac{\text{RAO}_j(k_{T,q}, c_{T,q}, \mu, \omega, \zeta)}{\text{RAO}_{j,\text{peak}}} \right] \right) \quad (35)$$

$$s.t. \begin{cases} c_{q,\min} < c_{T,q} < c_{q,\max} \\ k_{q,\min} < k_{T,q} < k_{q,\max} \end{cases}$$

where $\mu = m_c/m_s$ is the mass ratio between the WECDP and Spar, $\text{RAO}_{j,\text{peak}}$ is peak RAO of the Spar without the WEC, and ζ represents the wave direction. The $c_{q,\min}$ and $c_{q,\max}$ represent the minimum and maximum electromagnetic damping of each PTO in WECDP, and $k_{q,\min}$ and $k_{q,\max}$ represent the minimum and maximum stiffness of each PTO in

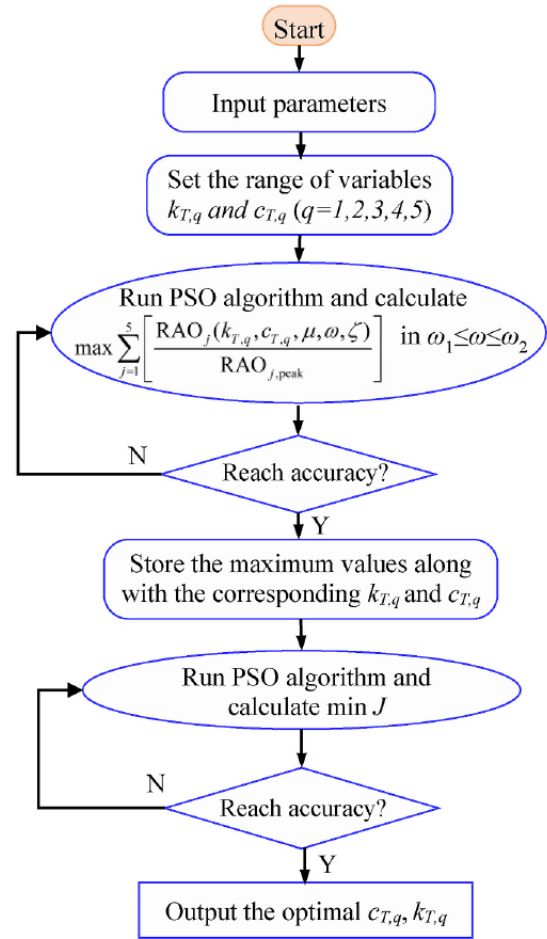


Fig. 22. Optimization method for multi-dimensional vibration control.

WECDP.

This study aims to mitigate the Spar's resonant risks (i.e., minimizing platform motion) by introducing a built-in WEC system. For this engineering optimization task, it is essential to consider multiple factors to meet practical requirements. In addition to the parameters mentioned above, factors such as multi-DOF motion phase differences, WEC cost, derived energy and weight of the WEC are discussed to provide readers with a clearer understanding of the proposed optimization objective. Specifically:

- (1) Regarding the "How the phase differences between the different movements were considered?"

In ocean environments, floating platforms are inevitably subjected to ocean waves, causing them to vibrate in multiple motion directions. Consequently, reducing the vibration of floating platforms is the primary focus of this study. Moreover, while the phase differences in different DOFs influence the vibration amplitude of the Spar, this potential relationship also suggests that the vibration amplitude can reflect the impact of phase differences in this study. It is foreseeable that incorporating phase differences into the optimization process would further complicate an already complex optimization procedure, and it may not facilitate subsequent analysis at the current conceptual design stage. Based on these considerations, phase differences are not explicitly included in this study.

- (2) Regarding the "How the WEC cost is considered in the optimization?"

The WEC cost includes the cost of manufacturing, installation, maintenance, and power transmission and transformation operation. However, these detailed economic cost parameters are relatively scarce because wave power generation technology is not yet fully mature, and successful commercial development has few cases. As a result, it is very difficult to calculate the investment costs of the WECs quantitatively. Historically, the cost of using sizable mechanical equipment is directly proportional to the scale of the device (Zhang et al., 2021; Orszaghova et al., 2022), which is well-known in the industry. For the same materials (i.e., same density), the mass of the device can be used as an approximation for its cost. This principle also applies to the development of WECs. This study uses the mass of the WEC to approximate its costs. This approach is relatively reasonable, especially during the conceptual design stage when detailed economic cost parameters are not yet available.

(3) Regarding the “Is the derived energy important in the optimization?”

The Spar platform supports numerous facilities and personnel and has significant value for exploring ocean resources. However, due to the effects of resonance, the potential excessive vibrations will cause platform’s structural damage or even lead to capsizing. Given the high construction and maintenance costs of offshore floating platform, minimizing operational risks caused by excessive vibrations as much as possible is the primary focus of this study. Following this principle, the derived energy by the WEC in this study is treated solely as an additional benefit of vibration control, and is not important in the optimization process. In our previous work (Huang et al., 2024), we have demonstrated that prioritizing derived energy significantly damages vibration control performance. Specifically, the vibration amplitude of the Spar floating platform increases significantly as more emphasis is placed on derived energy. In contrast, the gain in derived energy is relatively very small. Therefore, compared to vibration control, derived energy is not a priority in the optimization process of this study.

(4) Regarding “why the weight of the WEC is not included as an optimization variable?”

As a Spar’s vibration absorber, increasing the WEC’s weight can improve vibration control effectiveness by enabling it to accommodate more mechanical energy. This has been demonstrated in our previous research (Huang et al., 2024) as well as in other studies on mechanical vibration control (Bian and Jing, 2021; Bian, 2021). However, increasing the WEC’s weight inevitably raises the system’s cost. In

Table 4

The optimal PTO coefficients for mass ratio 0.05

PTO direction	Stiffness	Damping
Heave	1.507×10^4 N/m	1.998×10^4 Ns/m
Surge	1.819×10^3 N/m	6.964×10^3 Ns/m
Pitch	2.391×10^8 Nm/rad	6.009×10^7 Nms/rad
Sway	1.819×10^3 N/m	6.964×10^3 Ns/m
Roll	2.391×10^8 Nm/rad	6.009×10^7 Nms/rad

different engineering applications, there are distinct vibration control standards for floating platforms and budget constraints. Clearly, a trade-off between the WEC’s weight and its vibration control performance should be required, depending on the specific application needs. Consequently, it is not appropriate to use the WEC’s weight as the optimal variable. In view of this, we have set a range of WEC’s weight (i.e., mass ratios) to analyze the corresponding vibration control performance and provide a reference for practical design work (see section 5.2.3.).

5.1.2. Optimization method

As presented in Eq. (35), the optimization objective involves ten optimization variables specifically the stiffness $k_{T,q}$ and damping $c_{T,q}$ coefficients for $q = 1, 2, 3, 4, 5$. The target optimization wave frequency ranges from 0 rad/s to 1.2 rad/s. Thus, the traditional loop iteration method is unsuitable due to its disadvantages, such as long solution times and low accuracy. As a heuristic algorithm for multi-parameter optimization, particle swarm optimization (PSO) (Marini and Walczak, 2015) offers short search times and high-precision global search capabilities. Therefore, PSO is relatively appropriate for this multi-variable optimization task involving damping and stiffness. Fig. 22 presents the optimization procedure for multi-dimensional vibration control using PSO. The specific steps are as follows:

- Input the initial parameters for the system.
- Define the range of variables for stiffness ($k_{T,q}$) and damping ($c_{T,q}$) for each DOF direction ($q = 1, 2, 3, 4, 5$).
- Use the PSO algorithm to calculate the maximum $\sum_{j=1}^5 \left[\frac{RAO_j(k_{T,q}, c_{T,q}, \omega, \zeta)}{RAO_{j,peak}} \right]$ across the frequency range (ω_1 to ω_2).
- Verify if the accuracy requirement is met. If not, repeat the calculation in Step 3.
- If accuracy is met, save the maximum values and their corresponding stiffness ($k_{T,q}$) and damping ($c_{T,q}$) coefficients.

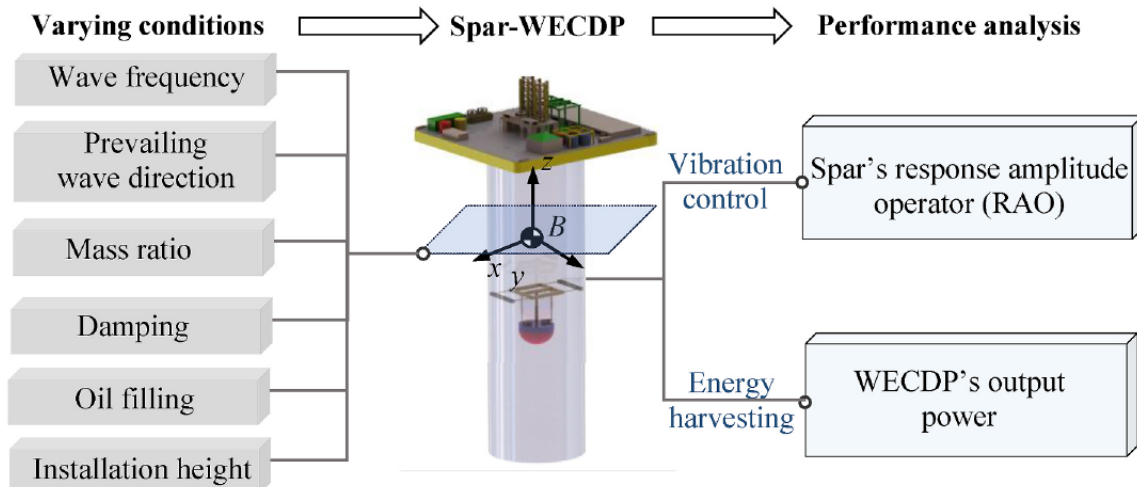


Fig. 23. Case description of performance analysis for Spar-WECDP.

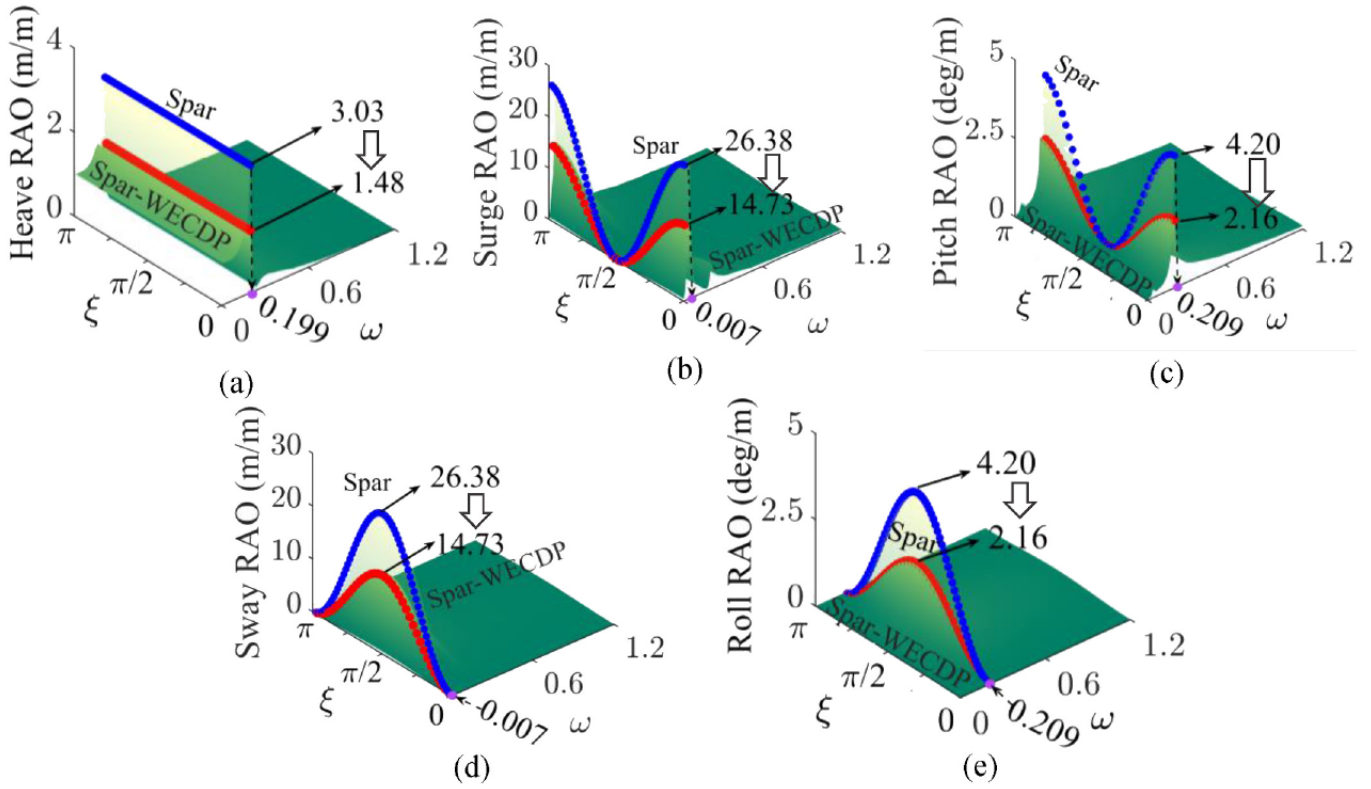


Fig. 24. Response amplitude operator of the Spar influenced by varying wave frequencies and directions in (a) heave, (b) surge, (c) pitch, (d) sway, and (e) roll directions.

Percentage reduction of RAO

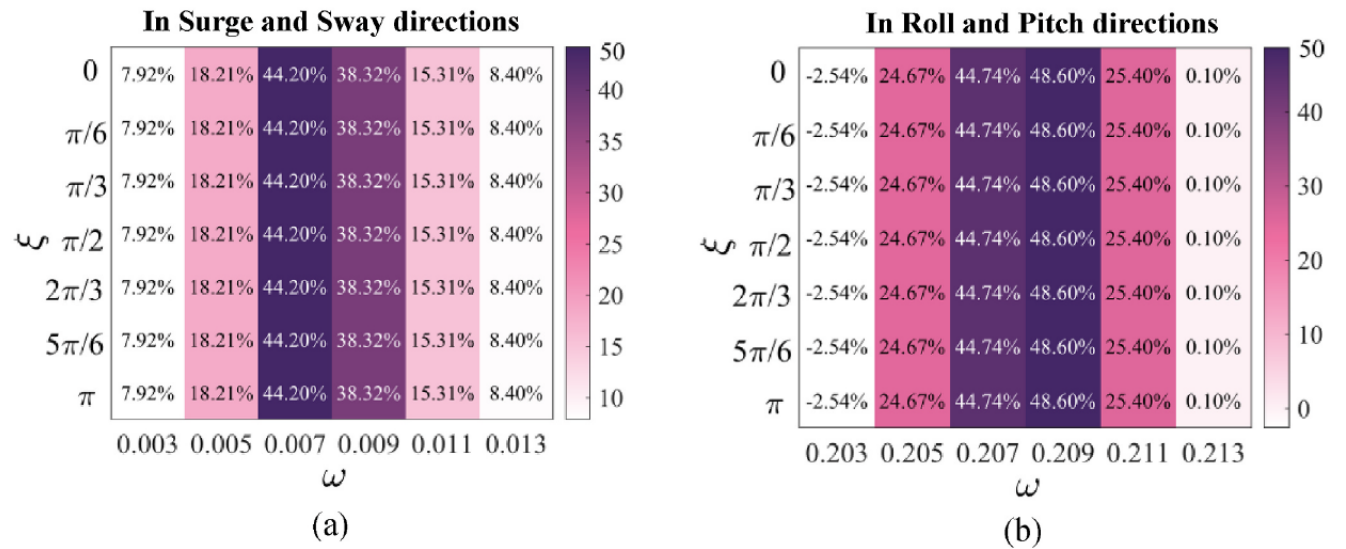


Fig. 25. The Spar's total response amplitude operator across rotation and translation directions: (a) percentage reduction in roll and pitch, and (b) percentage reduction in roll and pitch.

- vi. Apply the PSO algorithm again to minimize the objective function J .
- vii. Check if the accuracy criterion for minimizing J is satisfied. If not, repeat Step 6.
- viii. Once accuracy is achieved, output the optimized stiffness and damping coefficients.

5.2. Performance analysis under varying conditions

Wave frequency ω has been demonstrated to be a crucial factor affecting the motion characteristics of floating platforms (Zhang et al., 2023b; Dai, 2018; Liu, 2024). It should be pointed out that the operational location of the floating platform changes based on specific requirements and tasks. The prevailing wave directions ζ vary across different wave sites. In real-world engineering applications, the mass

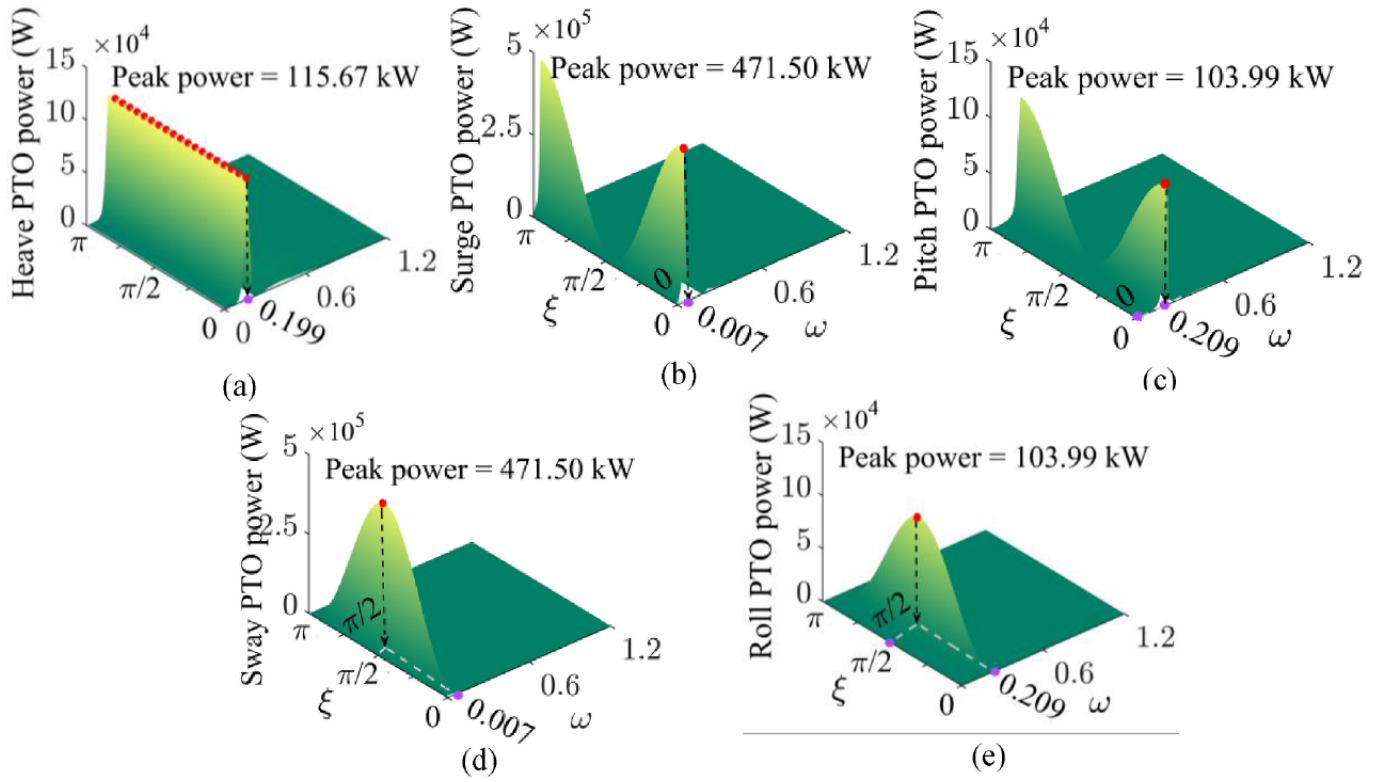


Fig. 26. The Spar-WECDP's output power under varying wave frequencies and directions: (a) Heave PTO, (b) Surge PTO, (c) Pitch PTO, (d) Sway PTO, and (e) Roll PTO.

ratio μ and damping can vary under different circumstances (Bian et al., 2022). Additionally, oil filling and the installation height of the WEC can also impact the proposed system. Based on the aforementioned optimization framework, this subsection examines the effects of these changes on vibration suppression and energy harvesting performance (see Fig. 23).

5.2.1. Vibration control performance analysis under varying wave states

In this case, the wave frequencies range from 0 rad/s to 1.2 rad/s, covering the frequency range near resonance that the Spar may encounter. The mass ratio μ equals 0.05. The prevailing wave directions are selected to range from 0 rad to π rad to further evaluate Spar-WECDP's application potential. Given that the prevailing waves typically constitute a significant portion of the wave conditions, we treat

ocean waves as unidirectional in the following analysis. This simplification helps reduce the complexity of calculations and design, while still providing a reasonable approximation that meets engineering requirements. For a mass ratio of 0.05, executing the proposed optimization procedure (see Section 5.1.2) in MATLAB 2022, the optimal PTO coefficients for stiffness and damping are as Table 4.

This case considers the wave direction ζ equal to 0 rad. At this angle, the Spar vibrates solely in the heave, surge, and roll directions because the wave direction is aligned with the x-axis. As shown in Fig. 24, the resonance peak frequencies for the Spar are different in the heave, surge, and roll directions. Nevertheless, under the influence of the single vibration absorber (i.e., the proposed WECDP), the Spar's resonance peaks in above three directions are effectively reduced by 51.2%, 44.2%, and 48.6%, respectively. This effectiveness is attributed to the independence

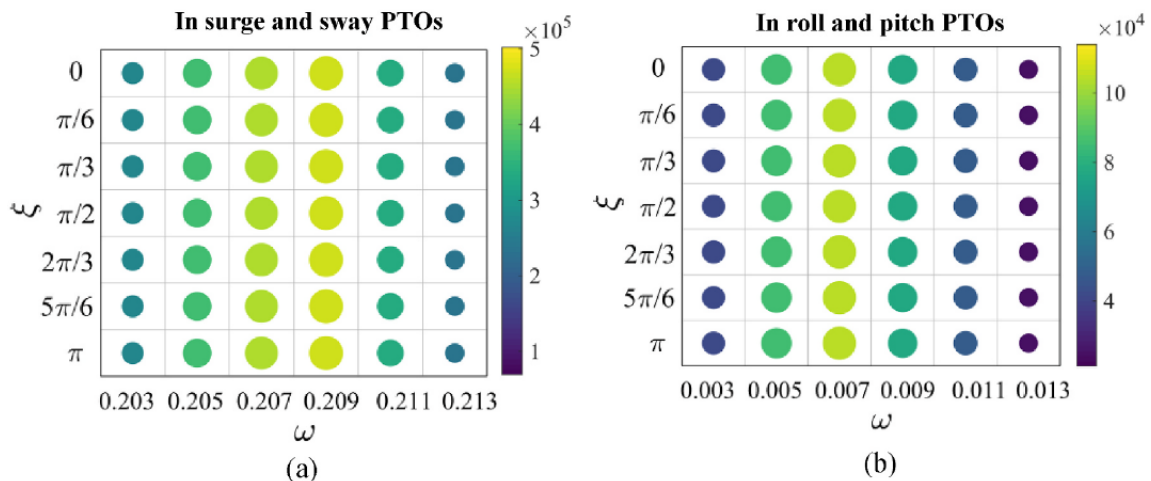


Fig. 27. The Spar-WECDP's output power in translation and rotation PTOs: (a) surge and pitch PTOs, (b) roll and pitch PTOs.

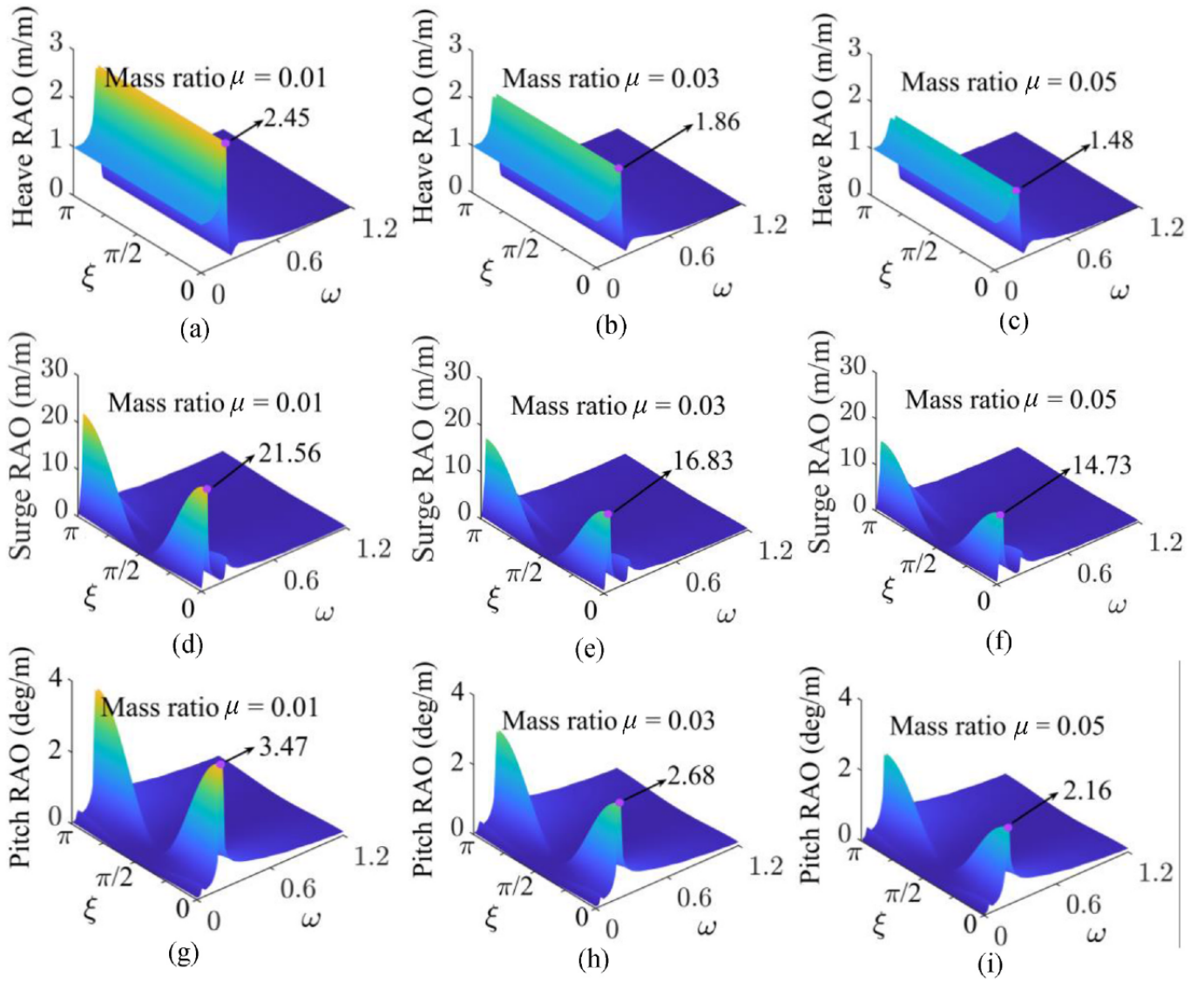


Fig. 28. The Spar floating platform's response amplitude operator under heave (a–c), surge (d–f), and pitch (g–i) directions at mass ratios $\mu = 0.01$, $\mu = 0.03$, and $\mu = 0.05$, respectively.

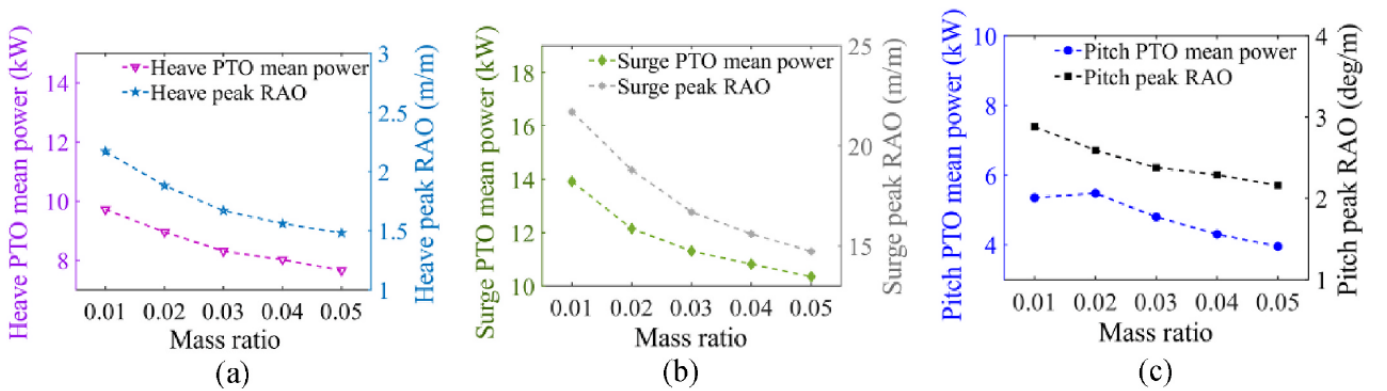


Fig. 29. The mean power and peak RAO of Spar-WECDP under different mass ratios: (a) in heave direction, (b) in surge direction and (c) in pitch direction.

of the PTOs within the built-in WECDP, allowing each PTO to absorb the Spar's vibration energy in its corresponding direction.

Moreover, the Spar's maximum RAO in the heave direction remains nearly constant at around 8.46, regardless of the variations in the wave

direction. However, the Spar's maximum RAO in the surge and pitch directions initially decreases as the wave direction changes and then increases. This phenomenon occurs because the change in the wave direction nearly does not affect the wave forces acting on the Spar in the

Table 5

The optimal PTO coefficients for mass ratio ranging from 0.01 to 0.04

Mass ratios	PTO direction	Stiffness	Damping
0.01	Heave	3.259×10^3 N/m	1.901×10^3 Ns/m
	Surge	3.932×10^2 N/m	6.603×10^2 Ns/m
	Pitch	5.169×10^7 Nm/rad	5.698×10^6 Nms/rad
	Sway	3.932×10^2 N/m	6.603×10^2 Ns/m
	Roll	5.169×10^7 Nm/rad	5.698×10^6 Nms/rad
0.02	Heave	6.390×10^3 N/m	5.298×10^3 Ns/m
	Surge	7.709×10^2 N/m	1.840×10^3 Ns/m
	Pitch	5.068×10^7 Nm/rad	7.939×10^6 Nms/rad
	Sway	7.709×10^2 N/m	1.840×10^3 Ns/m
	Roll	1.014×10^8 Nm/rad	1.588×10^7 Nms/rad
0.03	Heave	9.401×10^3 N/m	9.591×10^3 Ns/m
	Surge	1.134×10^3 N/m	3.331×10^3 Ns/m
	Pitch	1.491×10^8 Nm/rad	2.875×10^7 Nms/rad
	Sway	1.134×10^3 N/m	3.331×10^3 Ns/m
	Roll	1.491×10^8 Nm/rad	2.875×10^7 Nms/rad
0.04	Heave	1.229×10^4 N/m	1.455×10^4 Ns/m
	Surge	1.483×10^3 N/m	5.055×10^3 Ns/m
	Pitch	1.951×10^8 Nm/rad	4.362×10^7 Nms/rad
	Sway	1.483×10^3 N/m	5.055×10^3 Ns/m
	Roll	1.951×10^8 Nm/rad	4.362×10^7 Nms/rad

heave direction. Instead, due to the change in wave direction, the wave force and moment originally in the surge and pitch directions are redistributed to the sway and roll directions. Consequently, the maximum RAO of the Spar in the sway and roll directions exhibits contrasting trends compared with those observed in the surge and pitch directions (See Fig. 24(d) and (e)). Meanwhile, reductions of 44.2% and 48.6% in the Spar's resonance peaks also occur in the sway and roll directions due to the symmetry of the proposed built-in WECDP, consistent with the reductions observed in the surge and pitch directions.

To further analyze the impact of wave direction, the percentage reduction of RAO in the translation directions (surge and sway) and the rotation directions (roll and pitch) are investigated. As shown in Fig. 25, at any given wave frequency, the total percentage reduction of RAO in the surge and sway directions, as well as the roll and pitch directions, remains stable across different wave directions. This result indicates that the Spar-WECDP can achieve consistent vibration control regardless of the varying wave directions, further confirming the application potential of the proposed structure.

5.2.2. Energy harvesting performance analysis under varying wave states

This case analyzes the output power of the heave, surge, pitch, sway, and roll PTOs in the Spar-WECDP (See Fig. 26). Corresponding to the vibration reduction shown in Figs. 24 and 26 illustrates that the maximum output power for each PTO occurs at the resonant frequency of the respective motion direction of the Spar. The principle of energy

conservation indicates that changes in the vibration amplitude are directly related to changes in energy. Therefore, this notion implies that the WECDP can absorb the multi-dimensional vibration energy of the Spar with lower natural frequency. Meanwhile, this energy-harvesting property aligns with the natural motion characteristics of ocean waves, enhancing the practical usability of the proposed structure.

It is noted that the energy response spectra appear very narrow in Fig. 26. The reason lies in the resonance phenomenon of the Spar's motion. Specifically, through the optimization of the introduced WEC, it is observed in Fig. 24 that the vibration peaks of the Spar at resonant frequencies are significantly reduced. Since changes in vibration amplitude directly reflect changes in energy, this result confirms that the large amount of vibration energy at the resonant frequency is effectively transferred to the WEC. In contrast, the Spar's vibration amplitudes are small at other wave frequencies, which are clearly not the primary focus of optimization. As a result, compared to the resonant frequencies, the absorbed energy of the WEC at other frequencies is relatively much lower, causing the energy response spectrum to appear narrow.

After the calculation, the peak power values across various wave frequencies and directions are 115.67 kW, 417.50 kW, 103.99 kW, 471.5 kW, and 103.99 kW for the heave PTO, surge PTO, pitch PTO, sway PTO, and roll PTO, respectively. In combination with the significant reduction of the Spar's resonance peaks in Fig. 24, the above analysis results confirm that the proposed built-in WECDP effectively harvests the Spar's vibration energy across multiple motion directions, thereby achieving multi-dimensional vibration control. Meanwhile, this harvested energy can supply a portion of the power needed to operate equipment, support living facilities, and power scientific research instruments on the platform.

Interestingly, although the heave motion (3.03 m) has a significantly higher amplitude than the rotational motions (4.2 deg., i.e., 0.073 rad), the absorbed power from the rotational PTOs is relatively high compared to the heave PTO. This is due to the large difference in magnitude between the Spar's mass in heave motion and its moment of inertia in rotational motions (roll and pitch). Specifically, the Spar's moment of inertia in the rotational directions (roll and pitch) is 4.229×10^9 kgm², while its mass in heave motion is 7.466×10^6 kg (see Tables 1 and 2). This results in a difference of about three orders of magnitude. It is well known that the mechanical energy of the system in each DOF is proportional to the corresponding mass parameters. That is, the much larger moment of inertia in rotation causes the absorbed mechanical energy from the rotational dampers to appear relatively high.

To further analyze the impact of wave direction on output power, we investigate the Spar-WECDP's output power in the translation PTOs (surge and sway) and the rotation PTOs (roll and pitch). As shown in Fig. 27, under the same wave frequency, the total output power of the surge and sway PTOs remains constant across different wave directions.

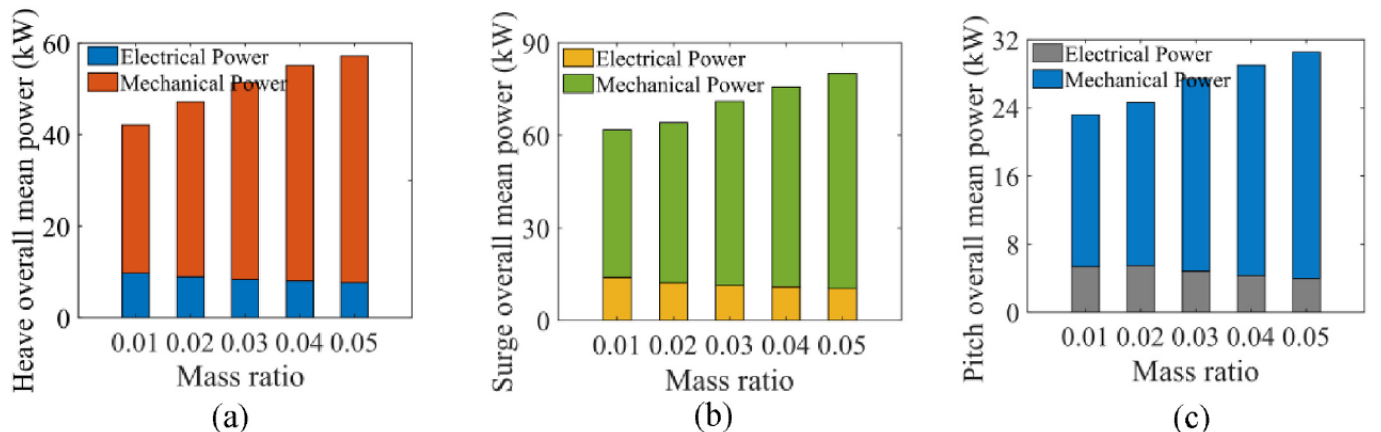


Fig. 30. The proportion of mechanical and electrical power under different mass ratios: (a) in heave direction, (b) in surge direction and (c) in pitch direction.

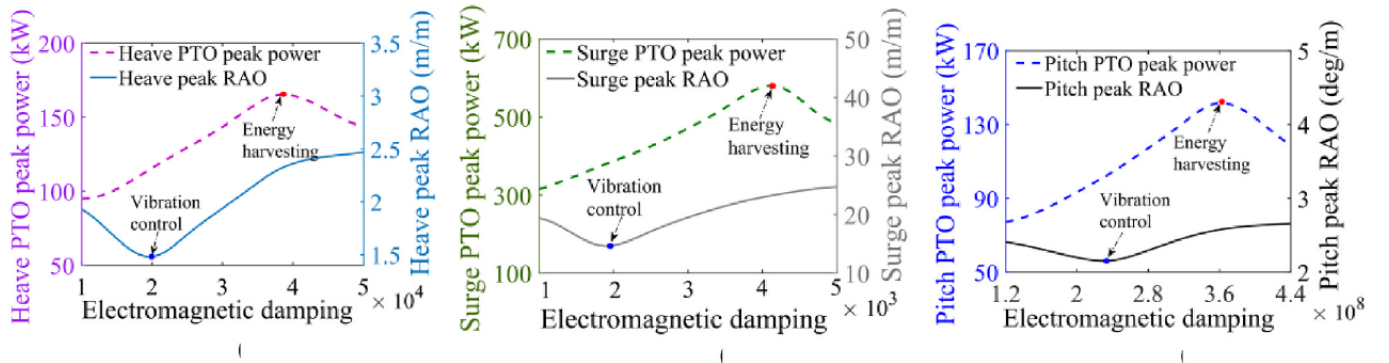


Fig. 31. The impact of damping changes on the Spar's peak RAO and the WECDP's peak power in each DOF direction.

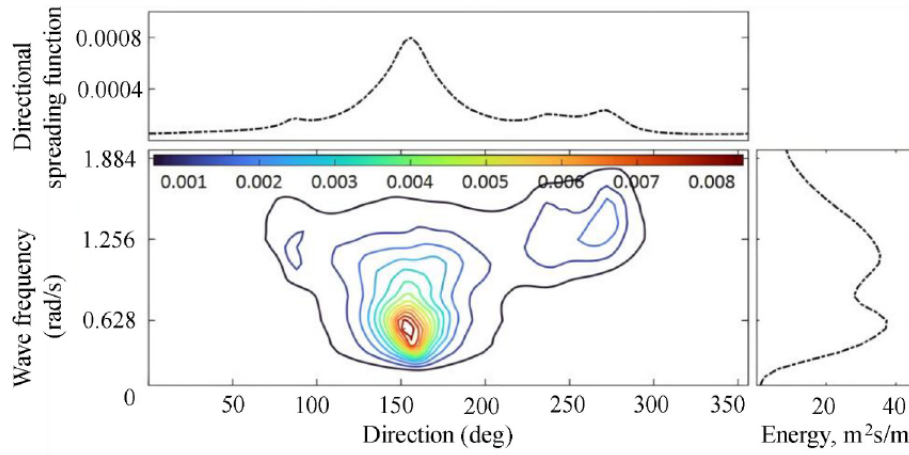


Fig. 32. Mean frequency-directional energy spectrum in the real Oman sea (Adibzade and Akbari, 2024).

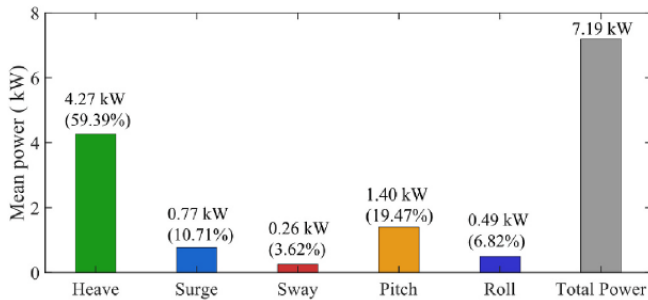


Fig. 33. The derived energy of WEC in different DOF directions under real Oman sea state.

The same observation is shown in the roll and pitch PTOs. Given that the output power in the heave PTO is direction-independent, these results indicate that wave direction has no effect on the total output power of the Spar-WECDP. Meanwhile, this notion also implies that the proposed Spar-WECDP has stable energy harvesting characteristics despite changes in the wave direction, which is highly desirable for practical applications.

5.2.3. Comprehensive analysis under varying mass ratios and damping

In this subsection, we further explore the influence of the different mass ratios and damping on the system performance (See Figs. 28 and 29). The optimal PTO coefficients for mass ratios of 0.01, 0.02, 0.03, and 0.04 are determined by executing the optimization procedure (see Section 5.1.2) and are presented in Table 5. Given the symmetrical performance characteristics of the proposed Spar-WECDP (as discussed in

Sections 5.2.1 and 5.2.2), we focus only on the performance in three directions: heave, surge, and pitch.

As shown in Fig. 28, with the mass ratio μ increases, the Spar's peak RAO decreases in each DOF direction. Although a higher mass ratio initially improves performance, its effect on vibration suppression becomes less pronounced with the further increase in the mass ratio. Considering that the manufacturing cost of a WEC is directly related to its mass (Dong et al., 2023; Albert et al., 2017), it is preferable to prioritize a WEC with a lower mass, provided it meets the vibration reduction requirements. In addition, since changes in vibration amplitude directly reflect changes in energy, this indicates that more of the Spar's vibration energy is transferred into the WEC system with an increasing mass ratio. Intuitively, the mean power in heave, surge, and pitch would be expected to increase with the mass ratio μ . However, except for a brief increase in the mean power of the pitch PTO, it is interesting to note that the mean power in these PTOs ultimately decreases slightly as the mass ratio μ increases.

The above phenomenon arises because the amount of energy harvested depends on the equivalent electromagnetic damping settings of each PTO in the WEC system. A reduction in vibration amplitude indicates that more vibration energy is being stored in the WEC system, but this does not necessarily mean it is being effectively converted into electrical power. Fig. 30 illustrates the proportion of mechanical and electrical power for each PTO in WEC. It shows that while the overall harvested energy of WEC increases with the mass ratio, a significant portion remains as mechanical power rather than being converted into electrical power. This explains the above counterintuitive trend observed.

Based on the obtained optimal PTO stiffness coefficients, we performed an analysis to further investigate the influence of damping

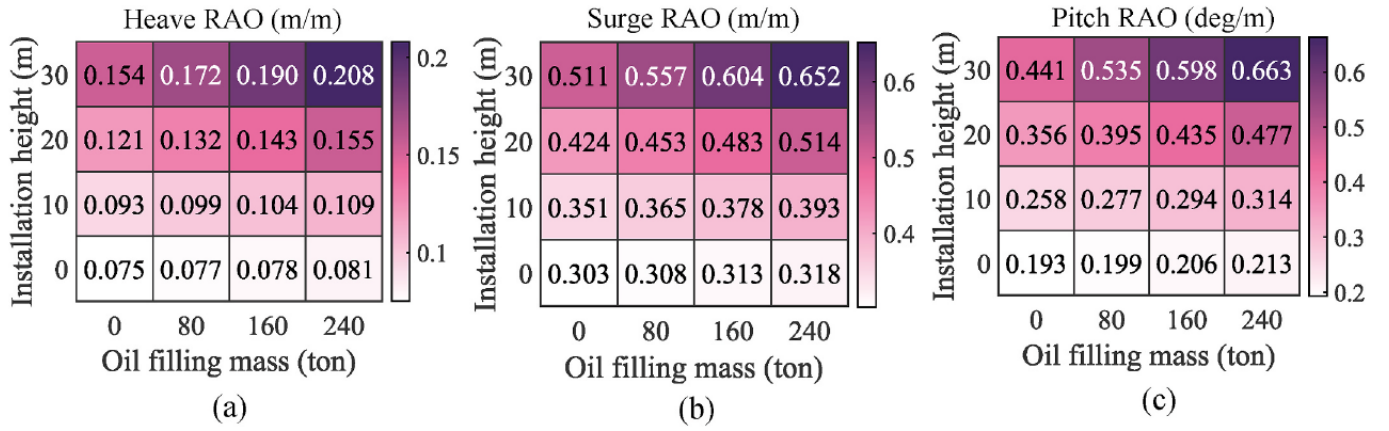


Fig. 34. Spar's RAO in different DOF directions affected by oil filling and WEC installation height: (a) in heave direction, (b) in surge direction and (c) in pitch direction.

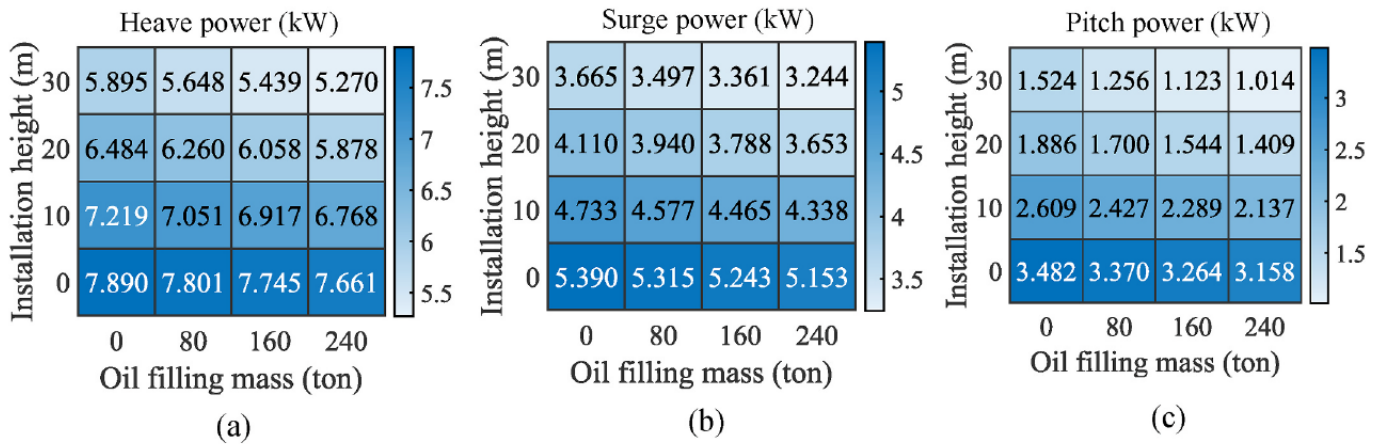


Fig. 35. WEC's power in different DOF directions affected by oil filling and WEC installation height: (a) heave direction, (b) surge direction, and (c) pitch direction.

changes on vibration control and energy harvesting under mass ratio equal 0.05. As shown in Fig. 31, when the damping deviates significantly from the optimal value (designed for vibration control), the Spar's peak RAO exhibits varying degrees of deterioration (i.e., RAO increases). This result is consistent with our earlier research (Huang et al., 2024; Bian, 2021) and other scholarly studies (Bian and Jing, 2021; Rashid and Nicolescu, 2008) in the field of structural vibration control. That is the effectiveness of vibration suppression in the Spar floating platform (equivalent to the primary structure) is significantly compromised when the damping of the WEC (equivalent to the vibration absorber) is detuned.

Fig. 31 also reveals that the output power in each DOF direction does not simply decrease with changing damping coefficients. Instead, it reaches a maximum power peak at a specific damping. This phenomenon is caused by the differing optimization objectives. Vibration control aims to transfer vibration energy into the WEC system rather than maximize its conversion to electric energy. In contrast, energy harvesting focuses on maximizing electric energy output. Consequently, the optimal damping coefficients for these two objectives differ.

5.2.4. Comprehensive analysis under real sea state

Large vibrations of the Spar occur primarily at its dangerous resonant frequencies, and significant energy can be harvested through effective vibration control at these frequencies (see Section 5.2.3). However, under real sea state conditions, the Spar experiences little to no movement most of the time. Additionally, in practical scenarios, the internal space of a Spar may be utilized as an oil storage reservoir, limiting the

available space for WEC installation. Considering these factors, this subsection first evaluates the energy harvesting performance of the proposed system under real sea state conditions. Second, it examines the effects of oil filling and the WEC's installation height.

(1) Analysis of energy harvesting performance under real sea state condition

Fig. 32 presents the mean frequency-directional energy spectrum in the real Oman sea based on measured data provided in the works of Adibzade and Akbari (Adibzade and Akbari, 2024; Adibzade, 2023). It can be observed that wave energy exhibits different distribution patterns across the frequency and directional domains. The majority of wave energy is concentrated around the 150° wave direction and a wave frequency of 0.628 rad/s.

Note: The wave direction 180° is defined as aligning with the Spar's positive x-axis.

Based on the above real sea state, we analyzed the derived energy of the WEC across different DOF directions, as shown in Fig. 33. Specifically, for the proposed WEC system, the mean power values under the Oman sea state are 4.27 kW for heave, 0.77 kW for surge, 0.26 kW for sway, 1.40 kW for pitch, and 0.49 kW for roll. The total derived energy is 7.19 kW, with the heave DOF contributing the most, accounting for 59.39%. This is followed by pitch (19.47%), surge (10.71%), roll (6.82%), and sway (3.62%).

In addition, accordingly the Spar's limit movement in real sea state condition, the total mean power in the real sea state is relatively small,

with 7.19 kW. However, it should be noted that once resonance occurs, the potential large vibrations of the Spar can significantly impact the performance of floating platforms and increase the risk of fatigue in structural components, possibly leading to overturning or sinking (Ma et al., 2020). Given the high construction and maintenance costs of offshore Spar floating platforms, minimizing operational risks caused by excessive vibrations is the primary concern. In line with this principle, this study aims to mitigate the Spar's resonant risks (i.e., minimizing platform motion) by introducing a built-in WEC system. This approach differs from traditional WEC systems, which aim to maximize motion to absorb wave energy. Moreover, the results in Section 5.2.1 also demonstrate the effective vibration suppression achieved by the proposed solution. Accordingly, the energy derived from the WEC system in this study is regarded as an additional benefit of vibration control rather than a primary objective. Based on above considerations, the relatively lower total derived energy under real sea states is acceptable within the scope of this study. Undeniably, we recognize that achieving an optimal balance between suppressing platform vibrations and enhancing energy harvesting is an important research direction. However, this aspect is beyond the scope of the current study and will be explored in future work.

(2) Analysis of the effects of oil filling and the WEC's installation height

Considering that the oil filling capacity has not been disclosed and the internal structural conditions are challenging to obtain due to commercial confidentiality, we made the following assumptions during the analysis: a) The mass of the oil filling is assumed to be 0 tons, 80 tons, 160 tons and 240 tons, with a density of 840 kg/m³ (Riazi and Eser, 2013); b) The WEC's installation height is assumed to range from 0 m to 30 m above the Spar's mass center, with an interval of 10 m.

To ensure comprehensive results, a cross-combination analysis of these two factors is performed to cover a wide range of scenarios. Based on the real Oman sea state, the most commonly observed specific wave condition (wave direction of 150° and a wave frequency of 0.628 rad/s) is selected for this analysis. Figs. 34 and 35 illustrate the variations in the Spar's RAO and WEC's power in different DOF directions influenced by these factors. Given the symmetrical performance characteristics of the proposed system structure, we focus only on the performance in three directions: heave, surge, and pitch.

It is evident that the WEC's installation height has a more significant impact on the system's performance compared to the oil filling mass. Specifically, as the WEC's installation height increases, the Spar's vibration amplitude in each DOF direction also increases. The most severe impact occurs in the pitch direction, where the Spar's peak RAO rises from 0.193 to 0.663 with an increase of up to 243.5%. Optimal vibration control effectiveness is achieved when the WEC's installation height is 0 m (i.e., positioned at the Spar's mass center). This phenomenon occurs because as the WEC's installation height deviates from the Spar's mass center, the surge PTO force application point no longer passes through its mass center but instead exhibits a certain offset. According to the force translation theorem, this offset generates an additional moment proportional to the distance from the mass center, significantly degrading the vibration control effectiveness in the rotational direction. Meanwhile, due to the chain reaction caused by this deterioration, the preset vibration suppression capability of the heave and surge PTOs in the WEC is inevitably reduced.

Since the aforementioned gradual weakening of vibration control effectiveness, the total absorbed energy that can be converted by the built-in WEC accordingly decreases. As a result, the derived energy in each DOF direction also shows a decreasing trend as the WEC's installation height increases (see Fig. 35). The most significant reduction occurs in the pitch PTO, with a decrease of up to 70.9%. Additionally, Figs. 34 and 35 show that an increase in oil filling mass worsens vibration control effectiveness to some extent and leads to a small

reduction in the derived energy of the WEC in each DOF direction. This is because the oil filling slightly alters the Spar's original mass and draft, thereby affecting its initial hydrodynamic design performance. It is important to emphasize that this study aims to mitigate the Spar's resonant risks (i.e., minimizing platform motion) by introducing a built-in WEC system. While the WEC's power output in the real sea state is relatively small, it is regarded solely as an additional benefit of vibration control rather than the primary objective.

6. Conclusions

The combination of the wave energy converters with the offshore floating platform presents a promising solution to achieve offshore floating platform's vibration control by energy harvesting. However, existing hybrid systems still face challenges in maintaining effective multi-dimensional vibration control under changing wave states. To bridge this research gap, with the classic Spar floating platform an example of floating platform, a novel Spar synergizing built-in wave energy converter with decoupled power take-offs is proposed. A detailed mathematical model that incorporates multiple physical domains is built and validated to reveal the operating mechanism. Based on this model, an optimization framework oriented to multi-dimensional vibration control is proposed, and the optimization results are analyzed. Noted that, a mass ratio below 5% is widely recognized as a practical upper limit to minimize costs, save space, and avoid significant impacts on the initial structural design. Following this guideline, the mass ratio in this study is set below 5%. Given this small mass ratio, it is reasonably assumed that its impact on the initial design of the Spar is negligible. The main conclusions can be drawn:

- (1) Through the synergy of built-in wave energy converter with decoupled power take-offs, the Spar's maximum response amplitude operators are effectively reduced by 51.2%, 44.2%, and 48.6% in the heave, surge, and pitch directions, respectively. Due to the symmetry of the proposed structure, the maximum response amplitude operator reductions for the sway and pitch are the same as that for the surge and roll.
- (2) In contrast with Spar's heave response amplitude operator, the surge, sway, pitch, and roll response amplitude operators vary with the change in the wave direction. Nonetheless, at any given wave frequency, the total percentage reduction of response amplitude operator in the surge and sway directions, as well as the roll and pitch directions, remains stable regardless of variations in wave direction.
- (3) The maximum output power of each power take-off occurs at the resonant frequency corresponding to the Spar's motion. This energy-harvesting property effectively utilizes the inherent low-frequency characteristics of large floating structures, making the proposed system better suited to the natural motion features of ocean waves and enhancing its practical usability.
- (4) At a given wave frequency, the total output power of the proposed system remains constant across different wave directions. This result demonstrates that the proposed structure's energy harvesting is direction-independent, which are highly desirable for practical applications.
- (5) The vibration control effectiveness of the proposed system improves slightly across all motion directions as the mass ratio μ increases. Nevertheless, a higher mass ratio μ results in increased wave energy converter construction expense. Consequently, it is more cost-effective to prioritize a wave energy converter with a lower mass, as long as it still meets the vibration control requirements.
- (6) Dampers control the spar vibrations and determine the total absorbed energy. The optimal damping values for vibration control and energy harvesting objectives are not the same.

- (7) The WEC's installation height significantly impacts the overall system performance compared to the oil filling mass. When the actual installed conditions are feasible, the WEC should be installed as close as possible to the Spar's mass center to achieve optimal performance.

In summary, this paper presents a solution for achieving multi-dimensional vibration control of the offshore floating platform by built-in wave energy converter with decoupled power take-offs, and conducts a numerical optimization investigation. It is undeniable that this analysis serves as the initial step toward realizing the proposed concept as a viable application. Future work will focus on prototype testing and further development of the power take-off controller structure.

CRediT authorship contribution statement

Yongxing Zhang: Writing – original draft, Visualization, Methodology, Conceptualization. **Zhicong Huang:** Project administration, Investigation, Funding acquisition. **Jing Bian:** Writing – review & editing, Writing – original draft, Supervision, Investigation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work is supported by the Science and Technology Planning Project of Guangdong Province under Grants 2023A0505050124 and 2023B0909050007, the Natural Science Foundation of Guangdong Province under Grant 2023A1515011623, the Guangzhou Municipal Science and Technology Bureau under Grant 2025A04J7064, and the Science and Technology Project of China Southern Power Grid Corporation under Grant 030000KC23040118.

References

- Aboutalebi, P., Garrido, A.J., Garrido, I., Nguyen, D.T., Gao, Z., 2024. Hydrostatic stability and hydrodynamics of a floating wind turbine platform integrated with oscillating water columns: a design study. *Renew. Energy* 221, 119824. <https://doi.org/10.1016/j.renene.2023.119824>.
- Adibzade, M., 2023. Spectral approach to evaluate multi-body floating wave energy converters in complex sea states. *Ocean. Eng.* 286, 115567.
- Adibzade, M., Akbari, H., 2024. Fully spectral approach to evaluate the performance of floating wave energy converters in directional complex sea states. *Ocean. Eng.* 306, 117999. <https://doi.org/10.1016/j.oceaneng.2024.117999>.
- Albert, A., Berselli, G., Bruzzone, L., Fanghella, P., 2017. Mechanical design and simulation of an onshore four-bar wave energy converter. *Renew. Energy* 114, 766–774.
- Anh, N.D., 2013. Design of TMD for damped linear structures using the dual criterion of equivalent linearization method. *Int. J. Mech. Sci.* 77, 164–170.
- Bao, X., Li, F., Sun, H., Iglesias, G., Shi, H., 2023. Performance characteristics and parameter analysis of a multi-DOF wave energy converter with hybrid power take-off systems. *Energy Convers. Manag.* 278, 116751.
- Bian, J., 2021. Analysis and Design of Nonlinear Damping and its Application. The Hoog Kong Polytechnic university. PhD thesis.
- Bian, J., Jing, X., 2020. Analysis and design of a novel and compact X-structured vibration isolation mount (X-Mount) with wider quasi-zero-stiffness range. *Nonlinear Dynam.* 101, 2195–2222. <https://doi.org/10.1007/s11071-020-05878-y>.
- Bian, J., Jing, X., 2021. A nonlinear X-shaped structure based tuned mass damper with multi-variable optimization (X-absorber). *Commun. Nonlinear Sci. Numer. Simul.* 99, 105829.
- Bian, J., Zhou, X., Ke, K., Yam, M.C., Wang, Y., 2022. Seismic resilient steel substation with BI-TMDI: a theoretical model for optimal design. *J. Constr. Steel Res.* 192, 107233.
- Bian, J., Zhou, X., Ke, K., Yam, M.C., Wang, Y., Qiu, Y., 2023. A passive vibration isolator with bio-inspired structure and inerter nonlinear effects. *Struct. Eng. Mech.* 88, 221.
- Blum, E.K., 1962. A modification of the Runge-Kutta fourth-order method. *Math. Comput.* 16, 176–187.
- Bruzzone, L., Fanghella, P., Berselli, G., 2020. Reinforcement learning control of an onshore oscillating arm wave energy converter. *Ocean. Eng.* 206, 107346.
- Chai, Y., Bian, J., Li, M., 2024. A novel quasi-zero-stiffness isolation platform via tunable positive and negative stiffness compensation mechanism. *Nonlinear Dynam.* 112, 101–123. <https://doi.org/10.1007/s11071-023-09018-0>.
- Chen, W., 2021. In: Efficiency Analysis of a 3-DOF Wave Energy Converter (SJTU-WEC) Based on Modeling, Simulation and Experiment.
- Chen, Z., 2024. Mechanical property identification and performance evaluation of a power take-off combined with a mechanical motion rectifier and a magnetic bistable device. *Appl. Energy*. <https://doi.org/10.1016/j.apenergy.2023.122091>.
- Cheng, Y., Fu, L., Dai, S., et al., 2022a. Experimental and numerical investigation of WEC-type floating breakwaters: A single-pontoon oscillating buoy and a dual-pontoon oscillating water column. *Coast. Eng.* 177, 104188.
- Cheng, Y., Dai, S., Dai, S., et al., 2022b. Energy conversion and hydrodynamic analysis of multi-degree-of-freedom wave energy converters integrated into a semi-submersible platform. *Energy Convers. Manag.* 252, 115075.
- Cheng, Y., 2024. Hydrodynamic investigation on a single-point moored offshore cage-wave energy converter hybrid system. *Ocean. Eng.* 299, 116848.
- Cheng, Y., Fu, L., Dai, S., Colliu, M., Cui, L., Yuan, Z., et al., 2022. Experimental and numerical analysis of a hybrid WEC-breakwater system combining an oscillating water column and an oscillating buoy. *Renew. Sustain. Energy Rev.* 169, 112909.
- Cummins, W., 1962. The impulse response function and ship motions. *Schiffstechnik* 9, 101–109.
- Dai, J., 2018. Modeling and investigation of load and motion characteristics of offshore floating wind turbines. *Ocean. Eng.* 159, 187–200.
- Dong, F., Pan, S., Gong, J., Cai, Y., 2023. Maximum power point tracking control strategy based on frequency and amplitude control for the wave energy conversion system. *Renew. Energy* 215, 118973. <https://doi.org/10.1016/j.renene.2023.118973>.
- Farahpour, H., Hejazi, F., 2023. Development of integrated semi-active adaptive vibration control system for bridges subjected to traffic loads. *Structures* 51, 1773–1794.
- Gao, H., Xiao, J., Liang, R., 2024. Capture mechanism of a multi-dimensional wave energy converter with a strong coupling parallel drive. *Appl. Energy* 361, 122828. <https://doi.org/10.1016/j.apenergy.2024.122828>.
- Gaspar, J.F., Hallak, T.S., Soares, C.G., 2018. Semi-submersible platform concept for a concentric array of wave energy converters. 8-10 2018 Lisbon Port. Adv. Renew. Energy. Offshore Proc. 3rd Int. Conf. Renew. Energy. Offshore RENEW, p. 307.
- Ghafari, H.R., Ghassemi, H., He, G., 2021. Numerical study of the Wavestar wave energy converter with multi-point-absorber around DeepCwind semisubmersible floating platform. *Ocean. Eng.* 232, 109177.
- Hemmati, A., Oterkus, E., Khorasanchi, M., 2019. Vibration suppression of offshore wind turbine foundations using tuned liquid column dampers and tuned mass dampers. *Ocean. Eng.* 172, 286–295.
- Hu, J., Zhou, B., Vogel, C., Liu, P., Willden, R., Sun, K., et al., 2020. Optimal design and performance analysis of a hybrid system combining a floating wind platform and wave energy converters. *Appl. Energy* 269, 114998.
- Huang, X., Yang, B., 2023. Towards novel energy shunt inspired vibration suppression techniques: principles, designs and applications. *Mech. Syst. Signal Process.* 182, 109496.
- Huang, Z., Zhang, Y., Bian, J., 2024. Offshore floating platform synergizing internally-installed self-reacting wave energy converters for optimizing vibration control and energy harvesting. *Ocean. Eng.* 313, 119429. <https://doi.org/10.1016/j.oceaneng.2024.119429>.
- John Nicholas, N., 1979. The theory of ship motions. *Adv. Appl. Mech.* 18, 221–283. [https://doi.org/10.1016/S0065-2156\(08\)70268-0](https://doi.org/10.1016/S0065-2156(08)70268-0).
- Jonkman, J., 2010. Definition of the floating system for phase IV of OC3. <https://doi.org/10.2172/979456>.
- Jr, C.R.S., Lima, F.K.A., 2016. Wind turbine and PMSG dynamic modelling in PSIM. *IEEE Lat Am Trans* 14.
- Kamarlouei, M., Gaspar, J.F., Calvario, M., Hallak, T.S., Mendes, M.J., Thiebaut, F., et al., 2020. Experimental analysis of wave energy converters concentrically attached on a floating offshore platform. *Renew. Energy* 152, 1171–1185.
- Kandasamy, R., Cui, F., Townsend, N., Foo, C.C., Guo, J., Shenoi, A., et al., 2016. A review of vibration control methods for marine offshore structures. *Ocean. Eng.* 127, 279–297.
- Ke, K., Bian, J., Zhou, X.-H., Yam, M.C., Wang, Y.-H., Gu, Z., et al., 2023. A novel prismatic-shaped isolation platform with tunable negative stiffness and enhanced quasi-zero stiffness effect. *Smart Struct. Syst. Int. J.* 31, 213–227.
- Kuo, C.-H., Dai, J.S., 2013. Task-oriented structure synthesis of a class of parallel manipulators using motion constraint generator. *Mech. Mach. Theor.* 70, 394–406. <https://doi.org/10.1016/j.mechmachtheory.2013.08.010>.
- Lamas-Pardo, M., Iglesias, G., Carral, L., 2015. A review of very large floating structures (VLFS) for coastal and offshore uses. *Ocean. Eng.* 109, 677–690.
- Lassioui, A., Fadil, H.E., Rachid, A., El-Idrissi, Z., Bouanou, T., Belhaj, F.Z., et al., 2020. Modelling and sliding mode control of a wireless power transfer system for BEV charger. *Int. J. Model. Identif.* 34, 171–186.
- Lin, R., Guo, W., Cheng, S.S., 2022. Type synthesis of 2R1T remote center of motion parallel mechanisms with a passive limb for minimally invasive surgical robot. *Mech. Mach. Theor.* 172, 104766. <https://doi.org/10.1016/j.mechmachtheory.2022.104766>.
- Liu, H.-F., 2024. Numerical investigation on the dynamic response of the semi-submersible aquaculture platform in regular waves. *Ocean. Eng.* 294, 116718.
- Ma, R., Bi, K., Hao, H., 2019. A novel rotational inertia damper for heave motion suppression of semisubmersible platform in the shallow sea. *Struct. Control Health Monit.* 26. <https://doi.org/10.1002/stc.2368>.
- Ma, R., Bi, K., Hao, H., 2020. Heave motion mitigation of semi-submersible platform using inerter-based vibration isolation system (IVIS). *Eng. Struct.* 219, 110833.

- Ma, R., Bi, K., Hao, H., 2021. Wave flume tests of a semi-submersible platform controlled by a novel rotational inertia damper. *Ocean. Eng.* 238, 109718.
- Ma, R., Bi, K., Zuo, H., Du, X., 2023. Inerter-based damping isolation system for vibration control of offshore platforms subjected to ground motions. *Ocean. Eng.* 280, 114726.
- Marini, F., Walczak, B., 2015. Particle swarm optimization (PSO). A tutorial. *Chemometr. Intell. Lab. Syst.* 149, 153–165.
- National Science & Technology Infrastructure. Measured Data. Available at: <http://mds.nmdis.org.cn/n.d>.
- Nguyen, H.P., Wang, C.M., 2020. Oscillating wave surge converter-type attachment for extracting wave energy while reducing hydroelastic responses of very large floating structures. *J. Offshore Mech. Arctic Eng.* 142, 042001.
- Oliveira-Pinto, S., Rosa-Santos, P., Taveira-Pinto, F., 2019. Electricity supply to offshore oil and gas platforms from renewable ocean wave energy: overview and case study analysis. *Energy Convers. Manag.* 186, 556–569.
- Orszaghova, J., Lemoine, S., Santo, H., Taylor, P.H., Kurniawan, A., McGrath, N., et al., 2022. Variability of wave power production of the M4 machine at two energetic open ocean locations: off Albany, Western Australia and at EMEC, Orkney, UK. *Renew. Energy* 197, 417–431. <https://doi.org/10.1016/j.renene.2022.07.053>.
- Ou, J., Long, X., Li, Q.S., Xiao, Y.Q., 2007. Vibration control of steel jacket offshore platform structures with damping isolation systems. *Eng. Struct.* 29 (7), 1525–1538.
- Park, G., 2022. Bilinear tuned mass damper for spar-type floating wind turbines. *Ocean. Eng.* 261, 112081.
- Pata, U.K., 2021. Renewable and non-renewable energy consumption, economic complexity, CO2 emissions, and ecological footprint in the USA: testing the EKC hypothesis with a structural break. *Environ. Sci. Pollut. Res.* 28, 846–861. <https://doi.org/10.1007/s11356-020-10446-3>.
- Ramachandran GKV, Robertson A, Jonkman JM, Masciola MD. Investigation of Response Amplitude Operators for Floating Offshore Wind Turbines: Preprint n.d.
- Rashid, A., Niclescu, C.M., 2008. Design and implementation of tuned viscoelastic dampers for vibration control in milling. *Int. J. Mach. Tool Manuf.* 48, 1036–1053.
- Rehman, S., Alhems, L.M., Alam, M.M., Wang, L., Toor, Z., 2023. A review of energy extraction from wind and ocean: technologies, merits, efficiencies, and cost. *Ocean. Eng.* 267, 113192.
- Riazi, M.R., Eser, S., 2013. Properties, Specifications, and Quality of Crude Oil and Petroleum Products. ASTM International.
- Rony, J.S., Karmakar, D., 2022. Coupled dynamic analysis of hybrid offshore wind turbine and wave energy converter. *J. Offshore Mech. Arctic Eng.* 144, 032002.
- Saeed, N., Gong, J., Wan, Y., Long, K., Saeed, A., Mei, L., et al., 2024. A novel design of multifunctional offshore floating platform structure based on topology optimization. *Eng. Struct.* 306, 117782.
- Sanchez, A.G., 2012. Dynamic model of wind energy conversion systems with PMSG-based variable-speed wind turbines for power system studies. *Int. J. Hydrogen Energy* 37, 10064–10069.
- Sharma, R., Kim, T.-W., Sha, O.P., Misra, S.C., 2010. Issues in offshore platform research-Part 1: semi-submersibles. *Int. J. Nav. Archit. Ocean Eng.* 2, 155–170.
- Si, Y., Chen, Z., Zeng, W., Sun, J., Zhang, D., Ma, X., et al., 2021. The influence of power-take-off control on the dynamic response and power output of combined semi-submersible floating wind turbine and point-absorber wave energy converters. *Ocean. Eng.* 227, 108835. <https://doi.org/10.1016/j.oceaneng.2021.108835>.
- Su, N., 2023. A novel lever-type inerter-based vibration absorber. *Int. J. Mech. Sci.* 254, 108440.
- Su, N., Bian, J., Peng, S., Chen, Z., Xia, Y., 2023a. Analytical optimal design of inerter-based vibration absorbers with negative stiffness balancing static amplification and dynamic reduction effects. *Mech. Syst. Signal Process.* 192, 110235.
- Su, N., Bian, J., Peng, S., Chen, Z., Xia, Y., 2023b. Balancing static and dynamic performances of TMD with negative stiffness. *Int. J. Mech. Sci.* 243, 108068.
- Su, N., Chen, Z., Xia, Y., Bian, J., 2024. Hybrid analytical H-norm optimization approach for dynamic vibration absorbers. *Int. J. Mech. Sci.* 264, 108796.
- Taghipour, R., Perez, T., Moan, T., 2008. Hybrid frequency-time domain models for dynamic response analysis of marine structures. *Ocean. Eng.* 35, 685–705. <https://doi.org/10.1016/j.oceaneng.2007.11.002>.
- Taghirad, H.D., 2013. Parallel Robots: Mechanics and Control. CRC press.
- Tao, J., Cao, F., Dong, X., Li, D., Shi, H., 2021. Optimized design of 3-DOF buoy wave energy converters under a specified wave energy spectrum. *Appl. Ocean Res.* 116, 102885.
- Tian, W., Wang, Y., Shi, W., Michailides, C., Wan, L., Chen, M., 2023. Numerical study of hydrodynamic responses for a combined concept of semisubmersible wind turbine and different layouts of a wave energy converter. *Ocean. Eng.* 272, 113824.
- Vicente, P.C., Falcão, A.F.O., Justino, P.A.P., 2013. Nonlinear dynamics of a tightly moored point-absorber wave energy converter. *Ocean. Eng.* 59, 20–36. <https://doi.org/10.1016/j.oceaneng.2012.12.008>.
- Wang, Y., Shi, W., Michailides, C., Wan, L., Kim, H., Li, X., 2022. WEC shape effect on the motion response and power performance of a combined wind-wave energy converter. *Ocean. Eng.* 250, 111038.
- Wei, Z., Shi, H., Cao, F., Yu, M., Li, M., Chen, Z., et al., 2024. Study on the power performance of wave energy converters mounted around an offshore wind turbine jacket platform. *Renew. Energy* 221, 119786.
- Wei-xing, C., Feng, G., Xiang-dun, M., 2018. Oscillating Body Design for A 3-DOF Wave Energy Converter, vol. 32.
- Wu, B., Lang, Y., Zargari, N., Kouro, S., 2011. Power Conversion and Control of Wind Energy Systems. John Wiley & Sons.
- Wu, H., Zhu, F., Yuan, Z., 2024. Effects of the WEC shape on the performance of a novel hybrid WEC-POWT system. *Energy* 288, 129907.
- Xu, J., Zhao, Y., Zhan, Y., Yang, Y., 2021. Maximum power point tracking control for mechanical rectification wave energy converter. *IET Renew. Power Gener.* 15, 3138–3150. <https://doi.org/10.1049/rpg.2.12213>.
- Yang, J., He, E.M., Hu, Y.Q., 2019. Dynamic modeling and vibration suppression for an offshore wind turbine with a tuned mass damper in floating platform. *Appl. Ocean Res.* 83, 21–29.
- Yang, B., Duan, J., Chen, Y., Wu, S., Li, M., Cao, P., 2024. A critical survey of power take-off systems based wave energy. *Ocean. Eng.* 298, 117149.
- Yao, J.T.P., 1972. Concept of structural control. *J. Struct. Div.* 98, 1567–1574. <https://doi.org/10.1061/JSDAAG.0003280>.
- Zhang, Y., 2023. Conceptual design and analysis for a novel parallel configuration-type wave energy converter. *Renew. Energy* 208, 627–644.
- Zhang, Q., He, J., Xu, Y., Hong, Z., Chen, Y., Strunz, K., 2016. Average-value modeling of direct-driven PMSG-based wind energy conversion systems. *Ieee T Energy Convers.* 37, 264–273.
- Zhang, B.-L., Han, Q.-L., Zhang, X.-M., 2017. Recent advances in vibration control of offshore platforms. *Nonlinear Dynam.* 89, 755–771. <https://doi.org/10.1007/s11071-017-3503-4>.
- Zhang, Y., Zhao, Y., Sun, W., Li, J., 2021. Ocean wave energy converters: technical principle, device realization, and performance evaluation. *Renew. Sustain. Energy Rev.* 141, 110764.
- Zhang, Z., Chen, X., Wu, H., Liu, W., Cui, L., 2023a. Numerical study of a novel hybrid system with the Wavestar wave energy converter array and a SPIC semi-submersible floating platform. *J. Clean. Prod.* 407, 137178.
- Zhang, H., Zhang, N., Cao, X., 2023b. Conceptualization and dynamic response of an integrated system with a semi-submersible floating wind turbine and two types of wave energy converters. *Ocean. Eng.* 269, 113517.
- Zhang, C., Li, D., Ding, Z., Liu, Y., Cao, F., Ning, D., 2024a. Wave energy converter with multiple degrees of freedom for sustainable repurposing of decommissioned offshore platforms: an experimental study. *Appl. Energy* 376, 124204. <https://doi.org/10.1016/j.apenergy.2024.124204>.
- Zhang, Y., Liu, Q., Gao, F., Zhou, S., Zhang, W., Chen, W., 2024b. Design and modeling of wave energy converter glider (WEC-Glider) with simulation validation in wave tank experiments. *Appl. Energy* 364, 123175. <https://doi.org/10.1016/j.apenergy.2024.123175>.
- Zhao, Y., Yuan, H., Zhang, Z., Gao, Q., 2024. Performance analysis and multi-objective optimization of the offshore renewable energy powered integrated energy supply system. *Energy Convers. Manag.* 304, 118232.
- Zhao, Y., Zhang, Y., Li, J., Jin, L., He, J., Zhang, X., et al., 2020. Inverse displacement analysis of a hyper-redundant bionic trunk-like robot. *Int. J. Adv. Rob. 17*, 1729881420903223.
- Zhou, B., Hu, J., Jin, P., Sun, K., Li, Y., Ning, D., 2023. Power performance and motion response of a floating wind platform and multiple heaving wave energy converters hybrid system. *Energy* 265, 126314.
- Zhou, Y., Liu, H., Kong, F., Wang, X., Jin, Y., Sun, C., et al., 2024. Research on the design and optimal control of the power take-off (PTO) system for underwater eel-type power generators. *Appl. Energy* 372, 123845. <https://doi.org/10.1016/j.apenergy.2024.123845>.
- Zhu, H., Hu, C., 2016. A study on control of wave energy converter for motion suppression of semisubmersible. *IFAC-PapersOnLine* 49, 380–385.
- Zou, S., Abdelkhalik, O., 2020. Collective control in arrays of wave energy converters. *Renew. Energy* 156, 361–369.