

Multi-degree-of-freedom decoupled mechanism for improving energy harvesting performance of wave energy converter

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ABSTRACT

The multi-degree-of-freedom wave energy converter (MDWEC) has attracted much attention due to its potential to efficiently harvest wave energy. However, current designs of MDWECs seldom consider coupled motions between the floating body and the power take-off (PTO), resulting in a loss of energy harvesting efficiency. To bridge this research gap, this paper explores the decoupled mechanism of active drive mechanical structures used for the MDWEC energy harvesting scenario. This decoupled mechanism enables the operating axes of PTOs to align one-to-one with the motion degree-of-freedom (DOF) of the floating body, thereby overcoming the energy harvesting efficiency limitations of the MDWEC caused by the coupled motion. To illustrate this idea, a novel multi-DOF decoupled wave energy converter (MDD-WEC) is presented according to real wave site conditions. A multi-physical-domain numerical model is developed. According to the Froude criterion, the reduced-scale prototype is constructed, and the numerical model test is performed in a wave tank. The proposed structure's motion response amplitudes obtained from wave tank experiments and numerical simulations under two irregular wave conditions showed relative errors below 5 %, confirming the numerical model's accuracy. Based on this, multi-PTO parameter analysis is subsequently performed to reveal the effects of different PTO configurations on the energy harvesting performance of the MDD-WEC. Benefiting from this characteristic, each PTO in the MDD-WEC can be independently optimized to absorb more wave energy according to wave conditions. The performance comparison results demonstrated that, compared with the point absorber WEC and the parallel configuration WEC with multi-DOF, the proposed MDD-WEC increases the capture width ratio (CWR) by 72.5 % and 39.3 % under regular wave conditions, respectively, and by 60.8 % and 32.9 % under irregular wave conditions.

1. Introduction

The large-scale use of fossil fuels supports rapid economic growth, but exacerbates energy shortages and environmental damage due to their high pollution and non-renewable nature [1]. As a renewable energy with great potential for development, ocean wave energy is very rich in storage, and developing wave energy resources using wave energy converters (WECs) with broad application prospects [2,3].

At present, numerous WECs with various structures and types have been developed by researchers worldwide. According to their operational mechanisms, these WECs can be categorized into oscillating body

WEC (OBWEC), oscillating water column WEC, and overtopping WEC. Among these, the OBWEC attracts continuous attention from countries and industries due to its feasibility and relatively high efficiency [4,5]. Based on the shape and size of the floating body and the direction of the incident waves, conventional OBWEC can be categorized into point absorber WEC (PAWECs) [6], terminator WEC [7], and attenuator WEC [8]. A common issue with these OBWECs is their low energy harvesting efficiency, which poses challenges for practical application [9].

To improve the energy harvesting performance of OBWECs, research efforts are mainly concentrated on three main avenues: optimization of geometric parameters, optimal control of the power take-off (PTO), and

Abbreviations: WEC, Wave energy converter; OBWEC, Oscillating body WEC; PAWEC, Point absorber WEC; PCWEC, Parallel configuration WEC; SCWEC, Series configuration WEC; PTO, Power take-off; DOF, Degree-of-freedom; MDWEC, Multi-DOF WEC; MDD-WEC, Multi-DOF decoupled WEC; CWR, Capture width ratio.

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improvements of operating mechanisms. The optimization of geometric parameters can effectively enhance the hydrodynamic performance of the floating structures within OBWECs, leading to increased wave energy absorption efficiency [10,11]. In the optimization process, maximizing power generation is one of the key optimization objectives. The optimization algorithms include genetic algorithms [12], direct search methods [13], and gradient-based techniques [14], all aimed at optimizing the floating body's geometric parameters. While these studies were lack of consideration on the influence of the multi-direction wave motions. The optimal designed device with a single degree-of-freedom (DOF) cannot maintain the high efficiency under complex wave conditions [15,16]. Apart from the geometric parameter optimization, the control of PTO can be optimized to further improve the overall wave energy harvesting performance. Specifically, this optimization method can dynamically tune the PTO parameters to synchronize the natural frequency of the OBWEC with the wave frequency, thereby improving energy harvesting through resonance effect [17]. At present, advanced PTO optimal control methods, including reactive control [18], latch control [19], and intelligent control [20,21], have been shown to enhance the wave energy harvesting performance of OBWEC to some extent. Nevertheless, similar to the drawbacks in optimization of geometric parameters, the energy harvesting performance of OBWECs obtained by tuning PTO parameters is restricted to a single DOF mode (e.g., heave, roll, or pitch), implying that these OBWECs capture wave energy from an only single direction.

To increase the energy harvesting potential of these OBWEC, some researchers developed multi-DOF WEC (MDWEC) concept [3,22,23]. This design conception is to enhance the wave energy harvesting capacity and the final output energy by increasing the motion DOFs of the floating body. Current configurations of MDWECs mainly include the series configuration WEC (SCWEC) and parallel configuration WEC (PCWEC) [3,24]. For instance, Chen et al. [25] developed a SCWEC with three DOFs in heave, surge, and pitch motion, which shows an excellent capture width ratio (CWR) under the complex wave condition. Using theoretical analysis and experimental tests, Zhang et al. [26] designed a multi-axis WEC capable of capturing wave energy across heave, surge, and sway DOFs. Considering the advantages of multi-DOF structures, Shi et al. [27] also designed a SCWEC with three DOFs (heave, surge, and pitch). They then developed the corresponding physical prototype and investigated the energy harvesting spectrum of the proposed WEC in different DOFs [28]. Through experimental tests, they demonstrated the proposed SCWEC's superior hydrodynamic characteristics and enhanced power absorption capabilities. Nevertheless, the poor structural stability of SCWECs in complex ocean environments has raised concerns regarding their practical application. In contrast, PCWECs offer not only multi-DOF features but also high structural strength and load-bearing capacity due to their parallel structure configuration. These features make them more suitable for operation in ocean environments [29]. Motivated by this idea, Gao et al. [30] designed a PCWEC capable of absorbing wave energy in the heave, surge, and pitch directions, and explored the influence of PTO damping on the energy harvesting performance in these directions. Recognizing the applicability of active drive parallel robot in the design of multi-DOF WEC, Galván-Pozos et al. [31] developed a novel PCWEC structure with multi floating body based on the Stewart-Gough parallel robot and analyzed its energy harvesting performance. In line with this idea, Aggidis et al. [32] presented a six-DOF PCWEC based on the parallel robot, which arranges multi-DOF movable components within a solid shell, enabling wave energy absorption in six DOFs. Considering the lack of specific structural design guidelines for PCWEC, we [33] systematically proposed a structure design idea to utilize parallel mechanisms for the development of MDWEC and explored its energy conversion performance. The research has shown that, compared with conventional OBWECs, the proposed structure exhibits a higher capture width ratio under varying wave conditions.

Admittedly, by increasing the DOF, MDWECs have enhanced the

energy harvesting potential of conventional OBWECs. However, these reported design schemes fail to consider the coupled motions between the floating body and the PTO in multiple DOFs, where the PTO operating axes do not align with the floating body's motion direction during the energy harvesting process. Through experimental tests, Tan et al. [34] systematically analysed this issue and demonstrated that the power output along one PTO axis can even act as a load on other PTOs due to this coupled motion, ultimately resulting in a loss of energy harvesting efficiency. Moreover, Tao et al. [35] also highlight this strong coupling motion characteristic for the parallel configuration of multi-DOF WECs. Considering the adverse effects of multi-DOF coupling motion, Zou and Abdelkhalik [36] present a time-varying linear quadratic control for the coupled pitch-surge modes, maximizing the energy harvested over a receding time horizon. However, this scheme does not essentially eliminate the coupling motion effects, which may result in incomplete energy harvesting under varying wave conditions.

To address this issue, this study explores the decoupled mechanism of active drive mechanical structures used for the MDWEC energy harvesting scenario. As illustrated in Fig. 1, the decoupled mechanism enables the operating axes of PTOs to align one-to-one with the motion DOFs of the floating body. In this way, the vibration energy of floating body in different DOF can be directly absorbed by corresponding PTOs, thereby overcoming the energy harvesting efficiency limitations of the MDWEC caused by coupled motion. The main contributions of this study are outlined below:

- (1) The structural decoupled mechanism for MDWEC is proposed. Based on this, a novel multi DOF decoupled WEC (MDD-WEC) is developed and constructed, providing a viable method to improve the energy harvesting performance under complex wave conditions.
- (2) A mathematical model of MDD-WEC integrating hydrodynamic, mechanical, and electrical aspects is developed and verified by wave tank test, providing guidance for modeling this type of multi-physical domain coupling.
- (3) The output power and capture width ratio of the MDD-WEC with multi-PTO configurations under different wave conditions are evaluated, providing in-depth understanding of the energy harvesting performance of the proposed MDD-WEC.

The structure of this manuscript is organized as: In Section 2, the specific procedures for the decoupled design of the MDD-WEC are presented. In Section 3, the multi-physical domain mathematical model is established, facilitating an analysis of its energy harvesting characteristics and providing a basis for subsequent parametric investigations. Section 4 constructs the reduced-scale prototype and validates the mathematical model through wave tank experiments. Based on this, Section 5 conducts parametric studies and performance comparisons under different wave conditions. Section 6 concludes the manuscript.

2. Structure design based on decoupled mechanism

To address the inadequate energy harvesting caused by coupled motion, this section provides a detailed design process for the MDWEC based on the decoupled mechanism.

2.1. Target wave site analysis and decoupled structure design

As shown in Fig. 2(a), the sea area addressed in this study is located at Paracel Island, China (coordinates: 16°40'14"N, 112°43'32"E). This region lies in the northwestern region of the South China Sea, surrounded by vast waters extending east, south, west, and north, offering favourable wave conditions.

Fig. 2(b) shows wave conditions based on a 15-year historical time series obtained from the China National Marine Data Center [37]. In the target sea region, wave heights range from 0.5 to 2.3 m, and wave

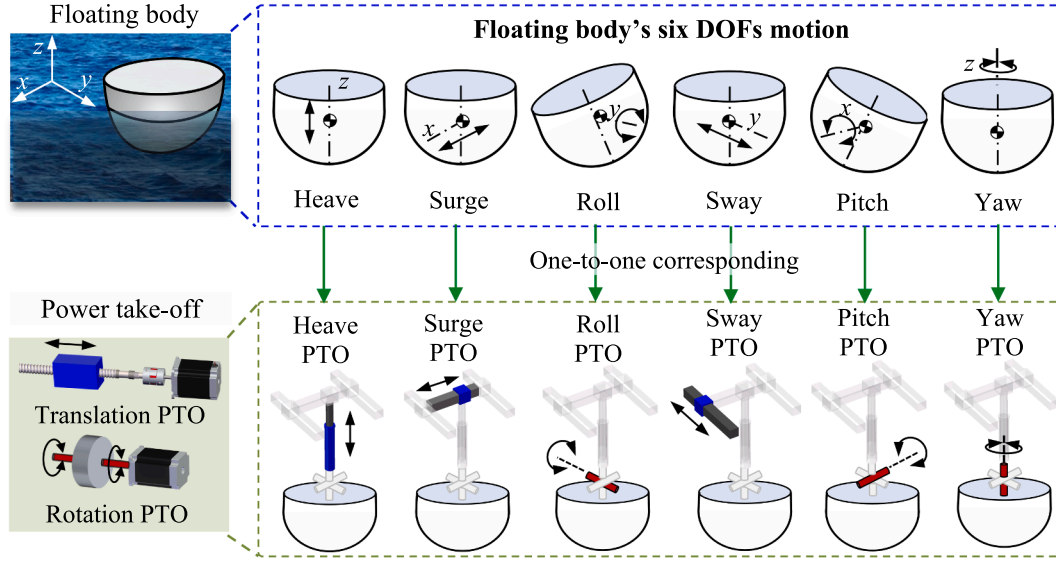


Fig. 1. Decoupled mechanism for multi-DOF WEC.

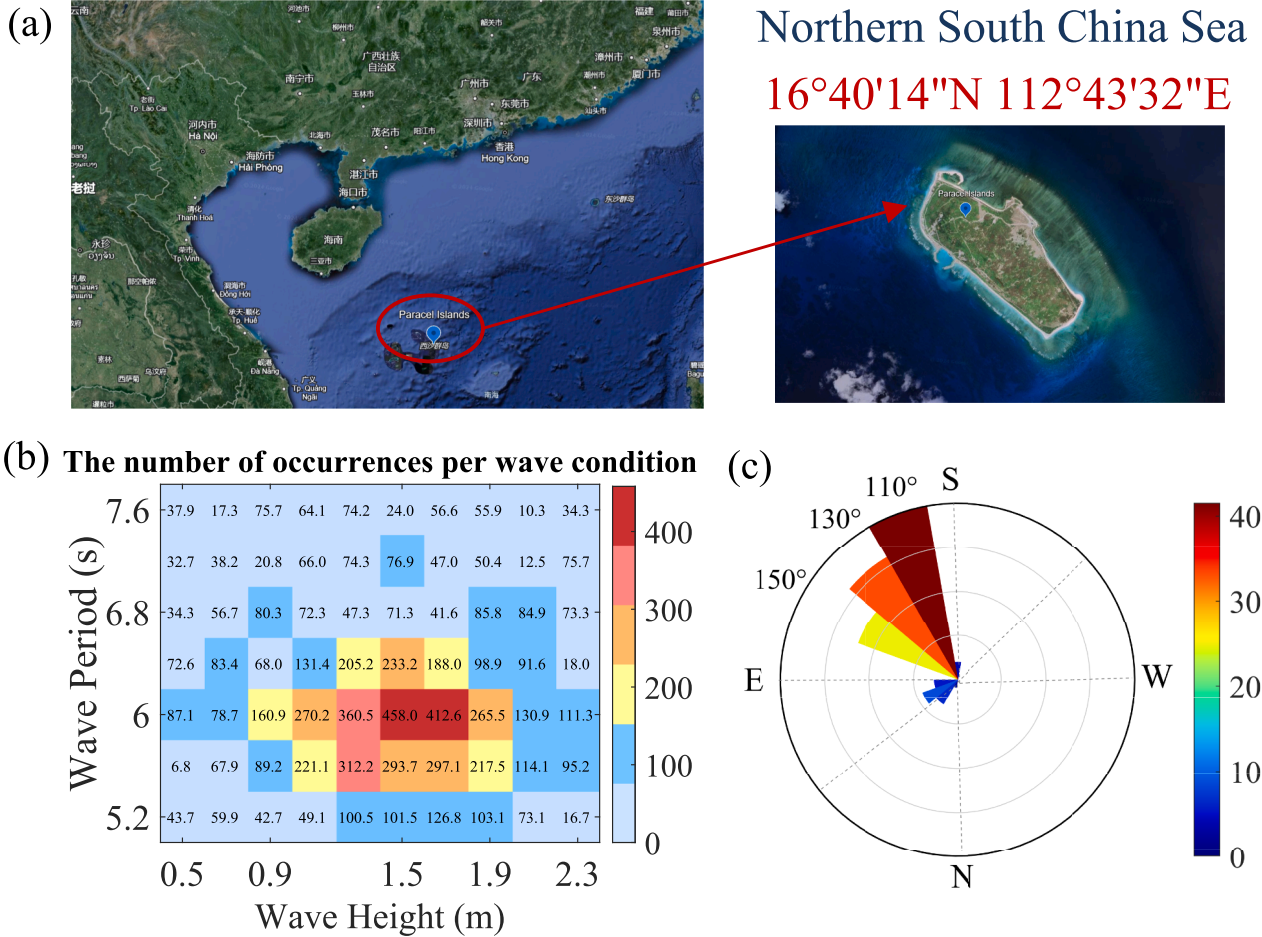


Fig. 2. Wave condition analysis of the target site based on 15 years of historical time series: (a) location of the target site, (b) analysis of wave height and wave period, (c) analysis of wave direction.

periods range from 5.2 to 7.6 s (i.e. wave frequencies ranging from 0.83 to 1.21 rad/s). The prevailing conditions are a wave height of 1.5 m and a period of 6 s. In addition, Fig. 2(c) indicates that although the incident wave directions cover a broad range from East to South-West by South,

the majority of the waves are concentrated within the sector from East by South to South by East ($110^{\circ} \sim 150^{\circ}$), accounting for 87.5 % of all wave directions. Given that the location is known to be swell-dominated, it is therefore reasonable to regard the wave field over an extended

period of time as unidirectional.

Theoretically, under above wave excitation, the floating body has up to six DOF motions: heave, surge, pitch, sway, roll and yaw. Utilizing the above all-motion can will make the energy harvesting more adequate. However, for extensively adopted the current axisymmetric floating bodies adhering to engineering practice standards, its have almost no motion in the yaw direction under a unidirectional ocean wave [25,38]. This means that, under above in wavefields of studied, the floating body with axisymmetric feature should have at most five motion DOFs of heave, surge, pitch, sway and roll. Based on the proposed decoupling mechanism (See Fig. 1), we provided the basic structure of the MDD-WEC in Fig. 3. The MDD-WEC comprises a floating body, multiple kinematic linkages, five PTOs and a fixed frame, with the operating axes of the five PTOs aligning one-to-one with the five motion DOFs of the floating body. When excited by ocean waves, the floating body oscillates, forcing the kinematic linkages to drive the PTOs and generate electricity.

2.2. Kinematic linkage design for decoupled mechanism

The multiple kinematic linkages connect the floating body to the fixed frame, with their structural properties define the functions achievable by the MDD-WEC. Currently, a variety of mechanical design methods, including graph theory, screw theory, displacement subgroup theory, linear translation, position and orientation characteristic set theory, and GF theory, are used to develop the structure of kinematic linkages [39]. Among these, GF theory [40] is recognized for its practicality, efficiency, and broad applicability, making it an ideal choice for designing MDD-WEC's kinematic linkages.

From a mechanical perspective, kinematic linkages can be classified into passive and active types [41,42]. The passive linkage imposes all necessary motion constraints on the end-effector, determining the entire system's DOF. In contrast, the active linkage functions as an unconstrained branch, ensuring that the end effector can achieve the required motion DOFs. Consequently, we choose a passive linkage with three translational and two rotational motions ($T_x T_y T_z; R_x R_y 0$) and an active linkage with three rotations and three translations ($T_x T_y T_z; R_x R_y R_z$) to satisfy the MDD-WEC's DOF requirements. To minimize failure risks and enhance structural reliability, the kinematic linkage is designed using only simple kinematic pairs, such as P (Prismatic pair), U (Universal pair), and S (Spherical pair), without incorporating complex kinematic pair structures. Leveraging GF theory, the linkage structure that incorporates these motion features can be effectively derived, as shown in Fig. 4.

2.3. Power take-off design

As a key component in power generation, the PTO converts mechanical energy into useful electrical energy. To enhance electric output, we design the translation PTO to match the heave, surge, and sway

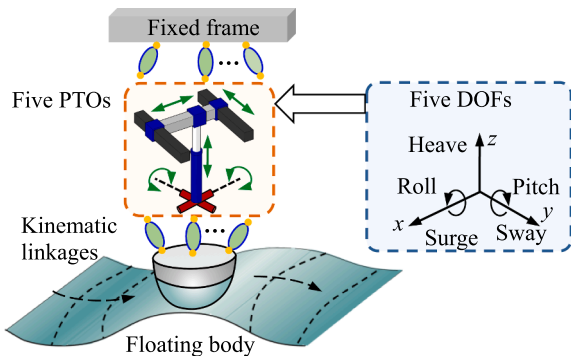


Fig. 3. Basic structure of the multi-DOF decoupled WEC.

motions, and the rotation PTO for the roll and pitch motions of the MDD-WEC. As shown in Fig. 5, the ball screw in the translation PTO converts linear motion into rotation, driving the permanent magnet synchronous generator (PMSG) to generate electricity. With the speed-increasing feature of the ball screw [43,44], the rotational speed derived from linear motion is amplified, better meeting the speed requirements of PMSG. Additionally, the planetary gearbox is used in the rotation PTO to transmit torque and amplify PMSG's speed [45,46].

Although the PMSG can generate electricity through the rotation of its shaft, the lower quality of electricity produced under real irregular wave conditions is not suitable for directly powering devices. In General, this electricity power needs to be stored in a high-capacity energy storage device, such as a battery or supercapacitor. As shown in Fig. 6, the alternating current (AC) generated by the PMSG is first converted into direct current (DC) through an AC/DC rectifier. It is then stabilized by a DC/DC converter to charge the battery.

2.4. Structure composition design

In the above sections, the kinematic linkage and PTO, which meet the decoupled operation mechanism, are designed. Combined with the basic structure of MDD-WEC shown in Fig. 3, the configuration model of MDD-WEC can be obtained and shown in Fig. 7 by using the axisymmetric installation method. It is noted that since floating body shapes are not the focus of the decoupled structure design in this paper, the floating body is designed as a commonly used hemispherical configuration, with the other shapes not discussed in detail.

The floating body parameters of the full-scale MDD-WEC model are listed in Table 1. Excited by the ocean waves, the designed MDD-WEC enables pitch and roll rotation and heave, surge, and sway movement within a specific range, aligning with wave motion characteristics and maximizing energy harvesting. In addition, its compact and axially symmetric design improves stability and supports diverse applications. As shown in Fig. 8, the MDD-WEC can be deployed on both fixed and floating platforms, supplying power for offshore facilities, accommodations, and research, while promoting wave energy adoption.

3. Mathematical modelling

To evaluate the proposed MDD-WEC's performance, this section constructs its motion response model under regular and irregular waves according to linear potential theory [23,24,47] and the Newton-Euler. Then, according to the operation mechanism of the designed PTOs, the dynamics including the permanent magnet synchronous generator (PMSG), power converter, and battery, are constructed to determines the output power of MDD-WEC.

3.1. Dynamic equations of MDD-WEC

Fig. 9 shows the structure sketch of the MDD-WEC. The xy plane of the fixed coordinate system $O-xyz$ is placed on the waterplane. The origin o of the coordinate system is at the centre of the waterplane, the x -axis is longitudinally along the floating body, the z -axis is oriented vertically upward, and the y -axis satisfies the right-hand rule. The $F-uvw$ is the moving coordinate system defined with the floating body's rotation center F . In still water, the u, v , and w axes are in the same direction as the x, y , and z axes.

The orientation between the local coordinate system $F-uvw$ and the fixed coordinate system $O-xyz$ is represented by two Euler angles. Thus, the rotation matrix ${}^O R_F$ can be defined as

$${}^O R_F = R_{rot(v_i)}(\varphi_i) R_{rot(u_i)}(\phi_i) = \begin{bmatrix} \cos\varphi_i & \sin\phi_i \sin\varphi_i & \cos\phi_i \sin\varphi_i \\ 0 & \cos\phi_i & -\sin\phi_i \\ -\sin\varphi_i & \cos\phi_i \sin\varphi_i & \cos\phi_i \cos\varphi_i \end{bmatrix} \quad (1)$$

The dynamic model of the MDD-WEC can be formulated by Newton-Euler method, which can be written as

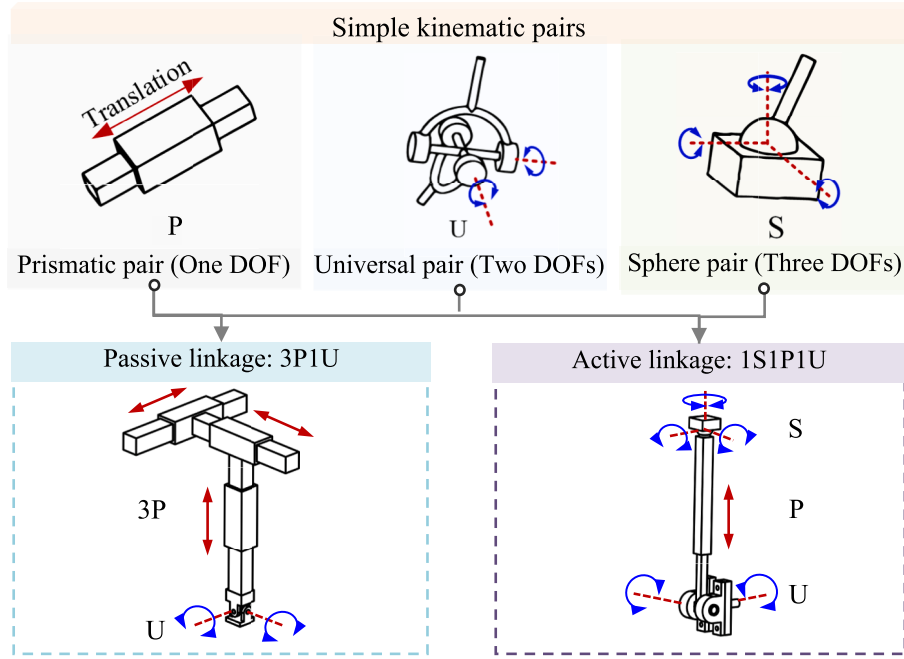


Fig. 4. The kinematic linkage design of MDD-WEC.

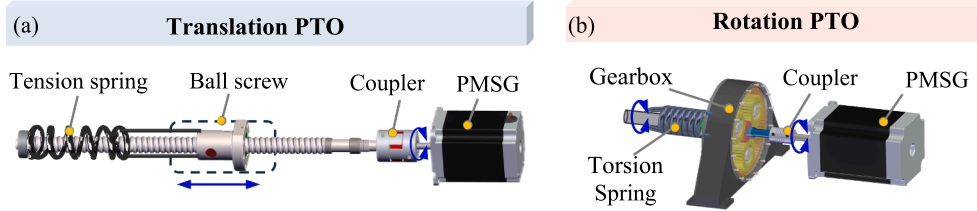


Fig. 5. Structure schematic of PTO: (a) Translation PTO, (b) Rotation PTO.

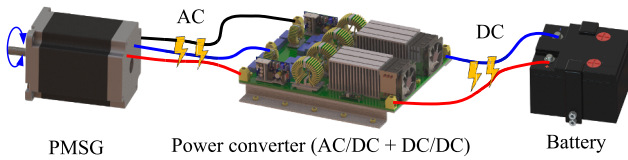


Fig. 6. Electrical conversion structure schematic.

$$\mathbf{M}_c \begin{bmatrix} \ddot{\mathbf{x}}_c \\ \ddot{\boldsymbol{\theta}}_c \end{bmatrix} + \mathbf{G}_c = \mathbf{F}_{wf} + \mathbf{F}_p \quad (2)$$

where $\mathbf{M}_c$ and $\mathbf{G}_c$ are the inertia item and gravity term of the MDD-WEC, respectively. $\ddot{\mathbf{x}}_c$ and $\ddot{\boldsymbol{\theta}}_c$ are the translation acceleration vector and rotation acceleration vector of MDD-WEC, respectively. $\mathbf{F}_{wf}$ and $\mathbf{F}_p$ represent the wave force and PTO force acting on the MDD-WEC, respectively.

Mathematical modeling relies on the structural and motion characteristics of the proposed MDD-WEC. The proposed system incorporates five degrees of freedom (DOFs): surge, sway, heave, pitch, and roll. As shown in Fig. 10, the architecture consists of two primary assemblies—a 2-DOF structure integrated with the surge and sway PTOs, and a 3-DOF

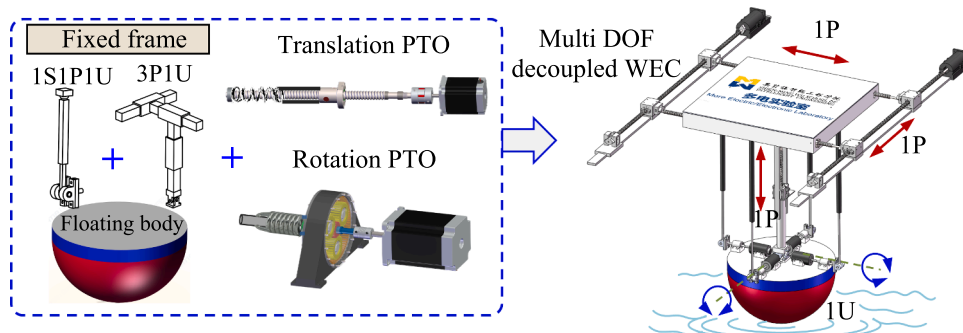


Fig. 7. The prototype synthesis of multi-DOF decoupled WEC.

Table 1
Floating body parameters of the full-scale MDD-WEC model.

Configuration	Item	Symbol	Value
Hemispherical floating body	Bottom radius	r_c	2.4 m
	Operating draft	D_{ra}	1.4 m
	Physical mass	m_c	11.904 t
	Inertia with respect to the x-axis	I_{xx}	1.889×10^4 kg $\cdot$ m 2
	Inertia with respect to the y-axis	I_{yy}	1.889×10^4 kg $\cdot$ m 2
	Inertia with respect to the z-axis	I_{zz}	2.819×10^4 kg $\cdot$ m 2
	Water depth	D_e	9 m

structure incorporating the heave, pitch, and roll PTOs. These two primary assemblies are connected through a white-top frame.

For the 3-DOF structure: as shown in Fig. 11, the 3-DOF structure adopts a typical parallel configuration with a passive linkage and four active linkages. These linkages connect the oscillating body to the top frame. All generators in the roll and pitch PTOs are fixedly installed in the floating body, whereas the generator in the heave PTO is fixedly installed in the top frame. The passive linkage imposes motion constraints on the end-effector (i.e., the floating body), defining the system's heave, roll, and pitch DOFs (see Section 2.2). In contrast, the active linkages function as unconstrained branches, enabling the end-effector to achieve the required three DOFs. The following presents a detailed analysis of the mass parameters associated with heave, roll, and pitch motion modes.

For the heave motion mode: As shown in Fig. 12, the heave motion of the floating body only causes the generators in the roll and pitch PTOs to move together with it, while the generator in the heave PTO remains unaffected. Consequently, the heave mode mass parameter in the MDD-WEC mass matrix is the total mass of the floating body and the generators mounted on it. For clarity, this study defines this total physical mass as m_c and uses it as the heave mode mass parameter in the MDD-WEC mass matrix.

For the pitch and roll motion modes: As shown in Fig. 13, the pitch and roll motions of the floating body only cause the generators in the roll and pitch PTOs to move together with it. Due to the fixed installation of the generators on top of the floating body, they can be regarded as a single entity. As a result, its overall moment of inertia is directly related to the pitch and roll mode mass parameter in the MDD-WEC mass matrix.

We further define the moment of inertia matrix about the mass center O of this entity as $I_c = \text{diag}[I_{roll}, I_{pitch}, 0]$, where I_{pitch} and I_{roll} represent the pitch and roll inertias, respectively. The zero in the matrix I_c represents the yaw inertia, which is omitted due to the absence of yaw motion in the MDD-WEC system (see the analysis in Section 2.1). Notably, as shown in Fig. 9, since the rotation center F of the proposed MDD-WEC does not coincide with its mass center O , the I_c needs to be transformed to the local coordinate system F - uvw through a coordinate transformation. Using ${}^O R_F I_c ({}^O R_F)^T$ which is widely applied in on

parallel structure coordinate transformations [48,49], we can obtain the moment of inertia matrix in the local coordinate system. Similar to the heave motion mode analysis above, the ${}^O R_F I_c ({}^O R_F)^T$ can be used to represent the pitch and roll mode mass parameter in the MDD-WEC mass matrix.

Furthermore, as shown in Fig. 14, the 2-DOF mechanical structure include two prismatic joints enable motion in the surge and sway directions. A translational PTO based ball screw is utilizing to convert MDD-WEC's linear motion (surge and sway) into rotation for the generator. Four translational PTOs are symmetrically arranged in an "I-shape" along the x and y axes, allowing the MDD-WEC to achieve surge and sway motions while driving the generator. Specifically:

For the surge motion mode: As shown in Fig. 15, due to the connection between the 3-DOF and 2-DOF structures, the MDD-WEC's surge motion involves the movement of the entire 3-DOF structure. Consequently, the surge mode mass parameter in the MDD-WEC mass matrix is the total physical mass of the 3-DOF structure. This comprehensive mass includes the floating body, top frame, motion linkages, and you mentioned the generators associated with the heave, roll, and pitch PTOs. For clarity, this study defines the total physical mass of the entire 3-DOF structure as m_{surge} , which serves as the surge mode mass parameter in the MDD-WEC mass matrix.

For the sway motion mode: As shown in Fig. 16, the MDD-WEC's sway motion includes not only the entire 3-DOF structure but also the surge PTO that contain the generator you mentioned. Consequently, the sway mode mass parameter in the MDD-WEC mass matrix is the sum of these contributions. For clarity, this study defines the total mass of the entire 3-DOF structure and the two surge PTOs as m_{sway} and uses it as the sway mode mass parameter in the MDD-WEC mass matrix.

In summary, the MDD-WEC inertia item M_c can be expressed as

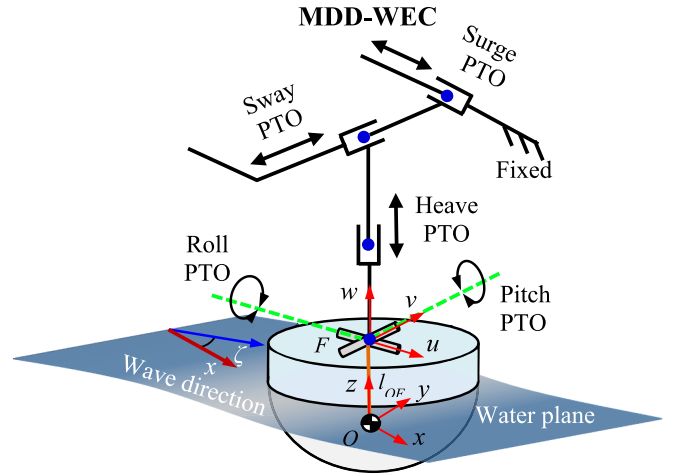


Fig. 9. Structure sketch of MDD-WEC.

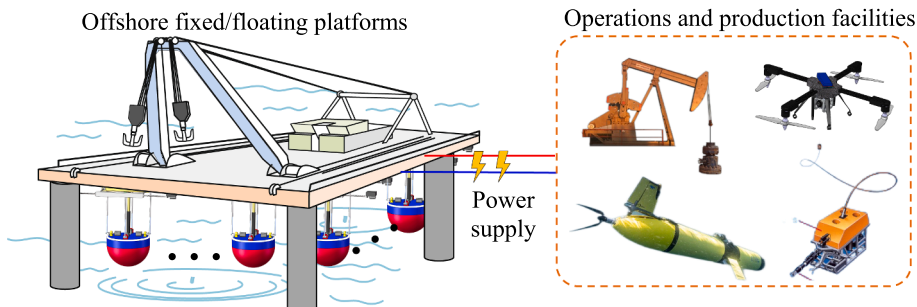


Fig. 8. The application scenario of MDD-WEC.

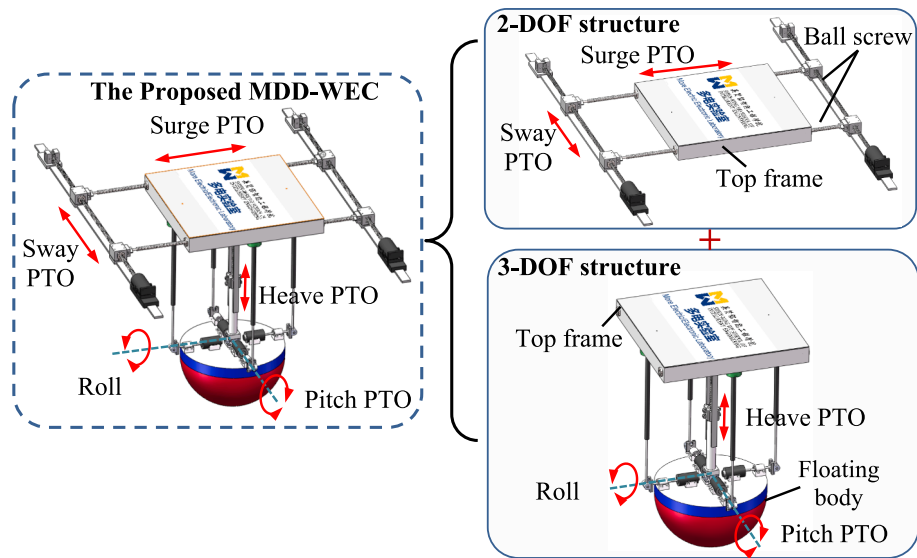


Fig. 10. The overall structure features for the proposed MDD-WEC system.

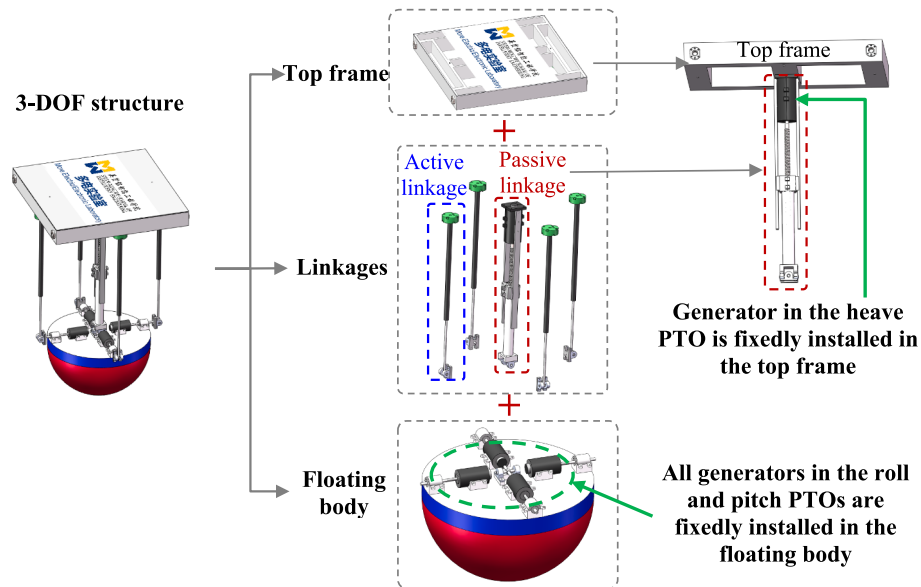
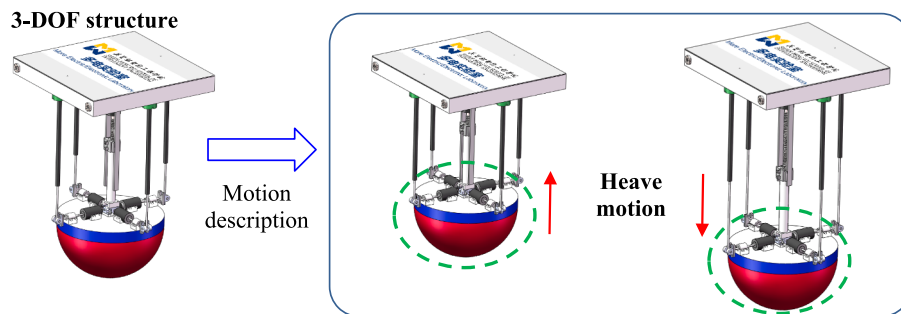


Fig. 11. Detailed configuration description of the 3-DOF structure.



Heave motion of the floating body only causes the generators in the roll and pitch PTOs to move together

Fig. 12. Motion analysis of the 3-DOF structure in the heave mode.

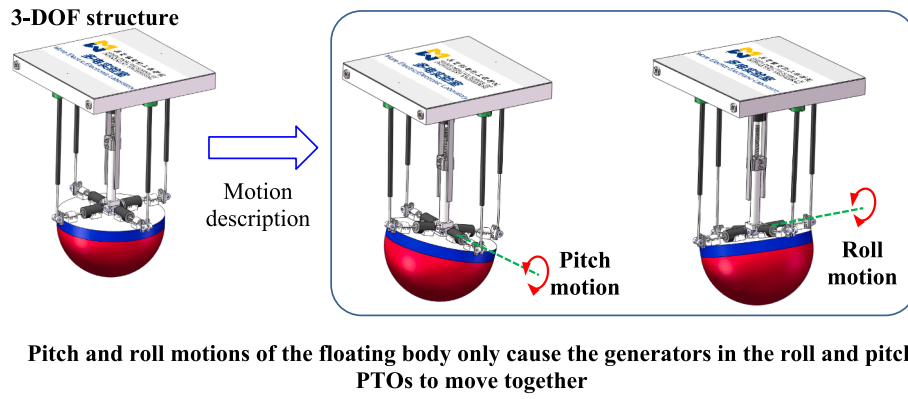


Fig. 13. Motion analysis of the 3-DOF structure in the pitch and roll modes.

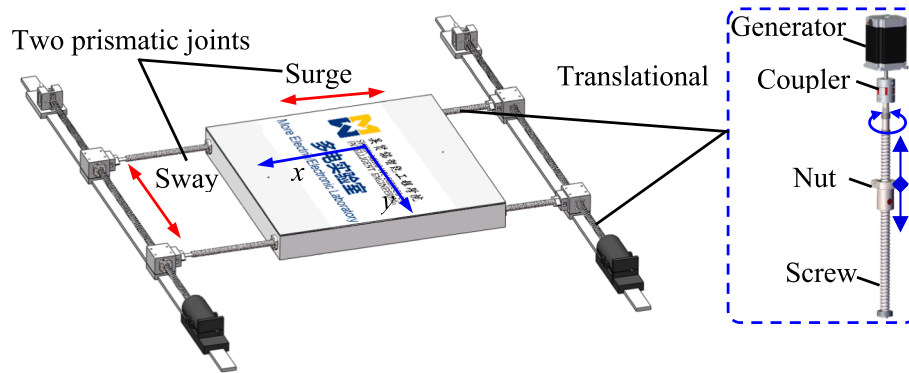


Fig. 14. Detailed configuration description of the 2-DOF structure.

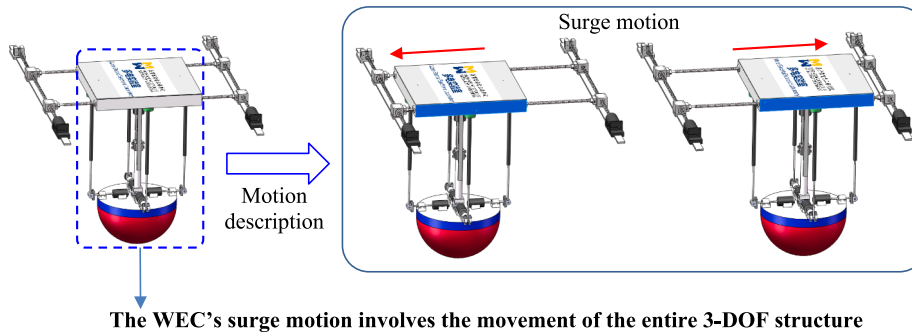


Fig. 15. Motion analysis of the 2-DOF structure in the surge mode.

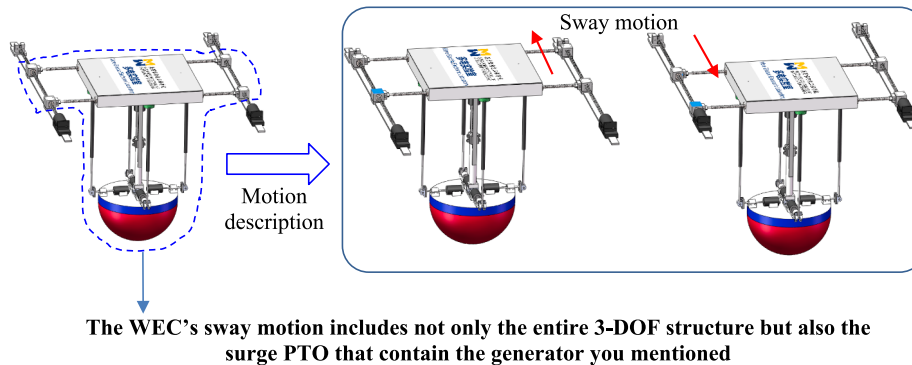


Fig. 16. Motion analysis of the 2-DOF structure in the sway mode.

$$\mathbf{M}_c = \begin{bmatrix} m_{\text{surge}} & 0 & 0 & & \\ 0 & m_{\text{sway}} & 0 & & \mathbf{0}_{3 \times 3} \\ 0 & 0 & m_c & & \\ & \mathbf{0}_{3 \times 3} & & {}^O\mathbf{R}_F \mathbf{I}_c ({}^O\mathbf{R}_F)^T & \end{bmatrix} \quad (3)$$

Noted that, in our mathematical modeling (see Fig. 9), the centroid of the MDD-WEC coincides with the origin O of the fixed coordinate system O -xyz, which is placed at the waterplane reference point. As a result, the inertia matrix $\mathbf{M}_c$ does not contain off-diagonal terms, which is consistent with multi-DOF wave energy converter modeling approaches in literature [35].

Within the O -xyz, fixed coordinate system, the gravity terms for MDD-WEC are expressed as

$$\mathbf{G}_c = [0 \ 0 \ m_c g \ 0 \ 0 \ 0]^T \quad (4)$$

The weight term $\mathbf{G}_c$ represents the inherent gravitational force acting on the WEC due to gravity. This term is typically treated as an external force in the dynamic equations, rather than as part of the WEC inertia matrix $\mathbf{M}_c$. Although the gravitational force is often neglected in some dynamic models due to the counteracting buoyancy forces from the ocean waves, this does not mean that the weight term is absent from the dynamic equations. Moreover, since this term only has a component along the vertical (z) axis within the world coordinate system, it does not involve any coupling between other degrees of freedom. This physical property also implies that the off-diagonal terms are not included in the weight term $\mathbf{G}_c$.

3.2. Wave forces acting on the MDD-WEC

The wave propagates along the ζ direction relative to the x -axis, and the wave force acting on the MDD-WEC is described as follows within the framework of potential flow theory.

$$\mathbf{F}_{wfj} = \mathbf{F}_{exc,j} + \mathbf{F}_{rad,j} + \mathbf{F}_{res,j}, \quad (j = 1, 2, 3, 4, 5) \quad (5)$$

where j represents the motion mode of the MDD-WEC, with 1, 2, 3, 4, 5 correspond to surge, sway, heave, roll, and pitch, respectively. $\mathbf{F}_{exc,j}$ represents the wave excitation force or moment in the j -mode, $\mathbf{F}_{rad,j}$ denotes the wave radiation force or moment in the j -mode due to the MDD-WEC's motion, and $\mathbf{F}_{res,j}$ is the hydrostatic restoring force or moment in the j -mode. Specifically,

(1) For wave excitation force or moment, they can be expressed under different wave conditions as follows [35,50].

$$\mathbf{F}_{exc,j} = \begin{cases} \frac{H}{2} \text{Re}\{F_{RAOj}(\omega_n)\} \cos(\omega_n t + \zeta_n + \theta_n) \cdots \text{Regular wave} \\ \sum_{n=1}^N \sqrt{2S(\omega_n)\Delta\omega} \text{Re}\{F_{RAOj}(\omega_n)\} \cos(\omega_n t + \zeta_n + \theta_n) \cdots \text{Irregular wave} \end{cases} \quad (6)$$

where H represents the regular wave height, $\text{Re}\{F_{RAOj}(\omega_n)\}$ is wave excitation force or moment modulus per wave amplitude in the j -mode for wave frequency ω_n , and θ_n represents the corresponding phase angle. An irregular wave is represented as the sum of multiple regular waves, with N denoting the number of regular wave components. The wave frequencies range from 0.100 rad/s and 3.141 rad/s, incremented by $\Delta\omega = 0.032$ rad/s. ζ_n and $S(\omega)$ are random phase angle and the spectral density of the irregular wave, respectively. As a result, the wave excitation force acting on MDD-WEC can be expressed as

$$\mathbf{F}_{exc} = [F_{exc,1} \ F_{exc,2} \ F_{exc,3} \ F_{exc,4} \ F_{exc,5} \ 0]^T \quad (7)$$

The above hydrodynamic parameters, including the modulus and phase of wave excitation force or moment, can be obtained by meshing the floating body model and solving it numerically in ANSYS AQWA [51,52]. For conciseness, these calculation results are provided in

Appendix A1.

(2) Based on Cummins equation [53], the radiation force $\mathbf{F}_{rad}$ can be expressed by convolution term

$$\mathbf{F}_{rad} = -\mathbf{A}(\infty) \begin{bmatrix} \ddot{\mathbf{x}}_s(t) \\ \ddot{\boldsymbol{\theta}}_s(t) \end{bmatrix} - \int_0^t \mathbf{K}(t-\tau) \begin{bmatrix} \dot{\mathbf{x}}_s(\tau) \\ \dot{\boldsymbol{\theta}}_s(\tau) \end{bmatrix} d\tau \quad (8)$$

where $\mathbf{A}(\infty)$ is the infinite-frequency added mass, $\mathbf{K}(t)$ denotes the retardation function used to describe the radiation damping force. These two terms are the critical to represent the radiation force.

For the infinite-frequency added mass $\mathbf{A}(\infty)$, it can be determined using frequency-domain identification in MATLAB, where the input is frequency and the output is added mass. These identification results are presented in Appendix A1 for brevity.

The retardation function $\mathbf{K}(t)$ represents a linear system. It should be noted that obtaining the retardation function $\mathbf{K}(t)$ through wave tank experiments is highly challenging. Instead, the use of commercial hydrodynamic software, such as ANSYS AQWA and WEC-SIM, provides a relatively high-fidelity alternative and is widely adopted in studies conducted by the National Renewable Energy Laboratory (NREL) [54]. Fig. 17 presents the detailed solution process.

Specifically, the geometric model of the floating body (including the rotating PTOs fixed on the floating body) is first established in 3D software based on its dimensions. This is followed by importing the model's structural parameters and generating the mesh. The still waterline and wave field conditions are then set within the ANSYS AQWA solver. The incident wave angle can be chosen from 0° to 180° with an interval of 22.5° . The wave frequency ranges from 0 rad/s to 20 rad/s during the calculation. Subsequently, the ANSYS AQWA solver is used to perform frequency-domain hydrodynamic diffraction analysis, which provides the wave excitation force/moment, added mass, and radiation damping. Finally, using the AH1 and LIS files [54] generated by the above frequency-domain analysis, the original formulation data of the retardation function $\mathbf{K}(t)$ is calculated through the boundary element method input/output (BEMIO) code from the WEC-SIM toolbox.

Based on this, the radiation damping term is typically approximated using the state-space model substitution approach [55]. For brevity, the detailed solution process is not provided in this paper.

$$\begin{cases} \int_0^t \mathbf{K}(t-\tau) \begin{bmatrix} \dot{\mathbf{x}}_c(\tau) \\ \dot{\boldsymbol{\theta}}_c(\tau) \end{bmatrix} d\tau = \mathbf{C}'\boldsymbol{\mu}(t) + \mathbf{D}' \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\boldsymbol{\theta}}_c(t) \end{bmatrix} \\ \dot{\boldsymbol{\mu}}(t) = \mathbf{A}'\boldsymbol{\mu}(t) + \mathbf{B}' \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\boldsymbol{\theta}}_c(t) \end{bmatrix} \end{cases} \quad (9)$$

where the $\boldsymbol{\mu}(t)$ represents the state variables of the state space model, and $\mathbf{A}'$, $\mathbf{B}'$, $\mathbf{C}'$, $\mathbf{D}'$ are constant coefficient matrices that describe the radiation damping force of the MDD-WEC. The fitting accuracy results of the state-space model are presented in Appendix A1 for brevity.

(3) The hydrostatic restoring force is zero in both the surge and sway directions. However, due to the influence of pitch and roll on the vertical restoring force, the hydrostatic restoring force in the heave direction is expressed as:

$$\mathbf{F}_{res,3} = \rho g \Delta V_d + k_{33}z \quad (10)$$

where g and ρ represent the acceleration of gravity and sea-water density, respectively. ΔV_d is the displaced volume change of the seawater, k_{33} is the hydrostatic stiffness in heave. z is the MDD-WEC displacement in heave direction. χ and β represent roll angle and pitch angle of MDD-WEC, respectively.

In the roll and pitch directions, the hydrostatic restoring moment [56] is expressed as

$$\mathbf{F}_{res,4} = k_{44}\chi \quad (11)$$

$$\mathbf{F}_{res,5} = k_{55}\beta \quad (12)$$

where k_{44} and k_{55} are the hydrostatic stiffness in roll and pitch,

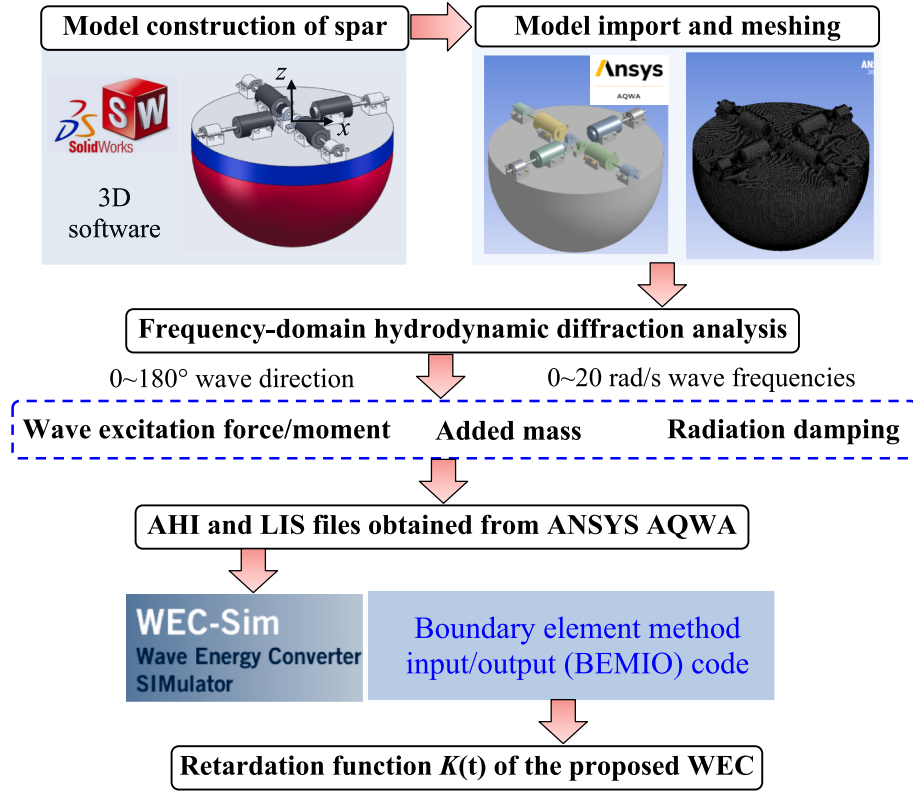


Fig. 17. Flowchart for calculating retardation function $K(t)$ of the proposed WEC using ANSYS AQWA and WEC-SIM.

respectively. Noted that, since the floating body in the WEC is fully axisymmetric in both the longitudinal and transverse directions, the off-diagonal terms in the hydrostatic stiffness matrix are zero according to the odd-function integral property. Consequently, the hydrostatic stiffness matrix can be expressed as $\mathbf{K}_{res} = \text{diag}(0, 0, k_{33}, k_{44}, k_{55})$, which is consistent with some references on the modeling of hydrostatic restoring forces for multi-DOF WECs [25,56].

As a result, the hydrostatic restoring force and moment acting on MDD-WEC can be expressed as

$$\mathbf{F}_{res} = [0 \quad 0 \quad F_{res,3} \quad F_{res,4} \quad F_{res,5}]^T \quad (13)$$

Noted that, the Eq. (13) describes the hydrostatic restoring force and moment of the WEC in different directions. This term is an external force

vector acting on the WEC rather than a matrix; therefore, it does not exclude any off-diagonal terms.

3.3. PTO force acting on the MDD-WEC

The PTO is an energy conversion device, and its force is directly related to its input velocity [57,58]. Specifically,

(1) For translation PTO, since the rotation center F of the MDD-WEC does not coincide with its mass center O , the linear velocity $\dot{\mathbf{x}}_c$ of MDD-WEC does not represent the PTO input velocity. According to the distance l_{OF} between points O and F , the linear velocity $\dot{\mathbf{x}}_F$ of the rotation center F can be obtained as:

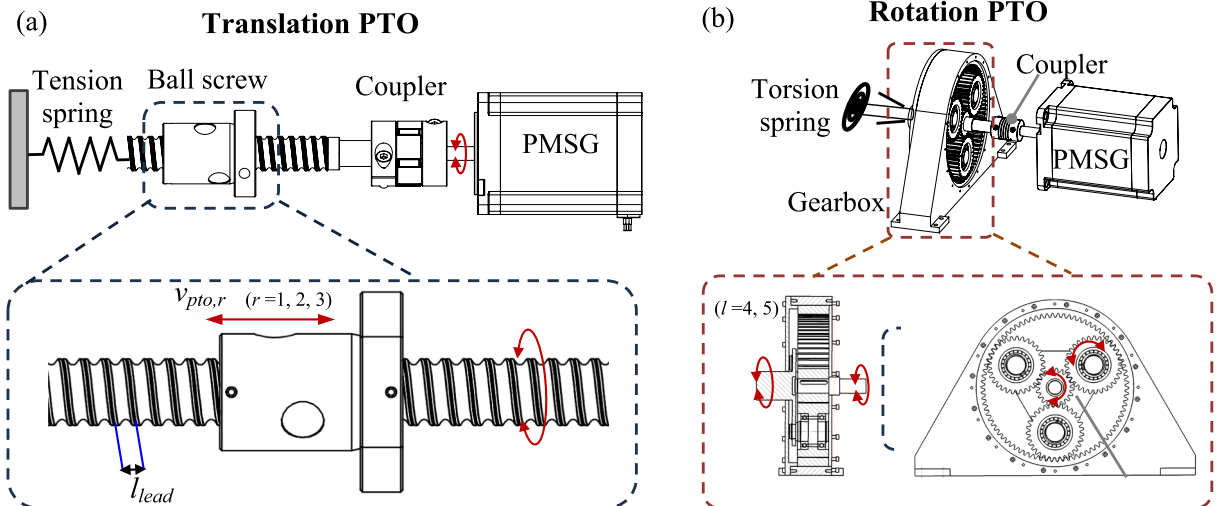


Fig. 18. The operation process of PTO: (a) Translation PTO, (b) Rotation PTO.

$$\dot{\mathbf{x}}_F = \dot{\mathbf{x}}_c + \dot{\boldsymbol{\theta}}_c \times [0 \ 0 \ l_{OF}]^T \quad (14)$$

Thus, the input linear velocity of translation PTO $[\dot{x}_{pto,1} \ \dot{x}_{pto,2} \ \dot{x}_{pto,3}]^T = {}^A\dot{\mathbf{x}}_F$. Where the subscripts 1, 2, and 3 represent the translation PTO in the surge, sway, and heave directions, respectively. As illustrated in Fig. 18(a), during the operation of the translation PTO, the coupler, ball screw and PMSG rotor only contain rotation. The tension spring provides stiffness for the translation PTO. Thus, the forces acting on the MDD-WEC from the translation PTO can be obtained in the O-xyz coordinate system.

$$\mathbf{F}_p = \begin{bmatrix} \sum_{r=1}^3 \mathbf{F}_{pto,r} \\ \sum_{l=4}^5 \mathbf{F}_{pto,l} \end{bmatrix} \quad (17)$$

3.4. Power output analysis

To obtain the motion response of the MDD-WEC, by substituting Eqs. (3)–(5), Eqs. (7)–(9), Eqs. (13) and (17) into Eq. (2), we can obtain

$$\begin{cases} [\mathbf{M}_c + \mathbf{A}(\infty)] \begin{bmatrix} \ddot{\mathbf{x}}_c(t) \\ \ddot{\boldsymbol{\theta}}_c(t) \end{bmatrix} + \mathbf{D}' \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\boldsymbol{\theta}}_c(t) \end{bmatrix} + \mathbf{C}'\boldsymbol{\mu}(t) = \mathbf{F}_{exc} + \mathbf{F}_{res} + \begin{bmatrix} \sum_{r=1}^3 \mathbf{F}_{pto,r} \\ \sum_{l=4}^5 \mathbf{F}_{pto,l} \end{bmatrix} - \begin{bmatrix} \mathbf{0}_{1 \times 2} \\ m_c \mathbf{g} \\ \mathbf{0}_{1 \times 3} \end{bmatrix} \\ \dot{\boldsymbol{\mu}}(t) = \mathbf{A}'\boldsymbol{\mu}(t) + \mathbf{B}' \begin{bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\boldsymbol{\theta}}_c(t) \end{bmatrix} \end{cases} \quad (18)$$

$$\mathbf{F}_{pto,r} = \left\{ \frac{2\pi}{\lambda l_{lead}} \left[c_{PMSG,q} \frac{2\pi}{l_{lead}} \dot{x}_{pto,r} + (I_{S,r} + I_{C,r} + I_{P,r}) \frac{2\pi}{l_{lead}} \ddot{x}_{pto,r} \right] + k_{T,r} \mathbf{x}_{pto,r} \right\} \begin{bmatrix} \delta_{1r} \\ \delta_{2r} \\ \delta_{3r} \end{bmatrix} \quad (15)$$

where $I_{S,r}$, $I_{C,r}$, $I_{G,r}$ and $\tau_{PMSG,r}$ are the rotational inertia of the ball screw, coupler, and PMSG rotor and PMSG's electromagnetic moment for the corresponding translation PTO, respectively. $k_{T,r}$ is the tension spring's stiffness. λ denotes the transmission efficiency of the translation PTO accounting for its operational friction. l_{lead} and δ denote the lead of the ball screw and Kronecker delta function, respectively. Considering the space limitations, the detailed derivation of Eq. (15) is provided in Appendix A2.

(2) For rotation PTO, its input rotation velocity is equal to the rotation velocity of MDD-WEC, i.e., $[\dot{\theta}_{pto,4} \ \dot{\theta}_{pto,5} \ 0]^T = \dot{\boldsymbol{\theta}}_c$; the subscripts 4, and 5 represent the rotation PTO in the roll and pitch directions, respectively.

As illustrated in Fig. 18(b), the rotation motion of the gearbox, coupler, and PMSG is involved in the rotation PTO's power generation process. The torsion spring provides stiffness for the rotation PTO. Similar to the translation PTO, the forces acting on the MDD-WEC from the rotation PTO can be expressed as follows. Considering the space limitations, the detailed derivation of Eq. (16) is provided in Appendix A2.

$$\mathbf{F}_{pto,l} = \left\{ \frac{1+\alpha}{\eta} \left[\tau_{G,l} + (I_{G,l} + I_{C,l} + I_{P,l}) (\ddot{\theta}_{pto,l} + \alpha \ddot{\theta}_{pto,l}) \right] + k_{S,l} \theta_{pto,l} \right\} \begin{bmatrix} \delta_{4l} \\ \delta_{5l} \\ 0 \end{bmatrix}, (l = 4, 5) \quad (16)$$

where $I_{G,l}$, $I_{C,l}$, $I_{P,l}$ and $\tau_{PMSG,l}$ represent the rotational inertia of the epicyclic gearbox, coupler, and PMSG rotor and PMSG's electromagnetic moment in the corresponding rotation PTO, respectively. $k_{S,l}$ is the torsion spring's stiffness, η denotes the transmission efficiency of the rotation PTO accounting for its operational friction, and α is the transmission ratio of gearbox. As a result, the PTO force acting on the MDD-WEC can be expressed as

The above differential equation set (18) can be solved by fourth-order Runge-Kutta method (RK4) [59], thus obtaining the motion of the MDD-WEC. The RK4 method is selected for its balanced trade-off between computational efficiency (fourth-order accuracy) and stability for nonlinear multi-DOF systems [60].

As is well known, the RK4 method has a global truncation error of $O(h^4)$, meaning that as the step size h approaches zero, the solution becomes increasingly accurate. To ensure convergence in our numerical simulations, we tested progressively smaller time steps ($h = \{0.01, 0.005, 0.001\}$ s) and verified whether further reduction in step size would result in relative errors remaining below 0.5 %. Based on these tests, a step size of 0.005 s is ultimately selected to confirm the stability and reliability of the used method.

Since the PMSG generates electricity as alternating current (AC), the generated AC must be converted into direct current (DC) through an AC/DC rectifier when storing electric energy. Then, it is stabilized by a DC/DC converter to charge the battery. To obtain MDD-WEC's output power, we present the equivalent circuit of the energy harvesting structure in Fig. 19 based on circuit theory. The rectifier (AC/DC) and power controller (DC/DC) are implemented through three-phase full-bridge passive circuits and three-phase interleaved parallel circuits, respectively. On the battery side, an open-circuit voltage source V_o in series with an internal resistance R_{int} models the battery's operating mechanism. When integrating circuit dynamics, the resistances of inductors, capacitors, and switching elements are generally small and are ignored in this modeling process.

According to the circuit impedance matching principle [61,62], the power converter and the battery can be equivalent to the load resistance of the PMSG. Thus, based on the PMSG equivalent model extensively studied in the literature [45,63–65], the electromagnetic moment can be expressed as follows.

$$\tau_{PMSG,q} = \frac{3k_e^2 \dot{\theta}_{g,q}}{6R_a + 4R_{L,q}}, (q = r, l) \quad (19)$$

where $R_{L,q}$ and $\dot{\theta}_{g,q}$ denote equivalent load resistance and mechanical angular velocity of the PMSG in the corresponding PTO, respectively. R_a and k_e represent stator phase resistance, and voltage constant of PMSG, respectively. Noted that, the PMSG electromagnetic moment $\tau_{PMSG,q}$ is equal to the product of the equivalent electromagnetic damping $c_{PMSG,q}$ and the angular velocity $\dot{\theta}_{g,q}$ of the PMSG (see Eq. (15d) and Eq. (16d) in

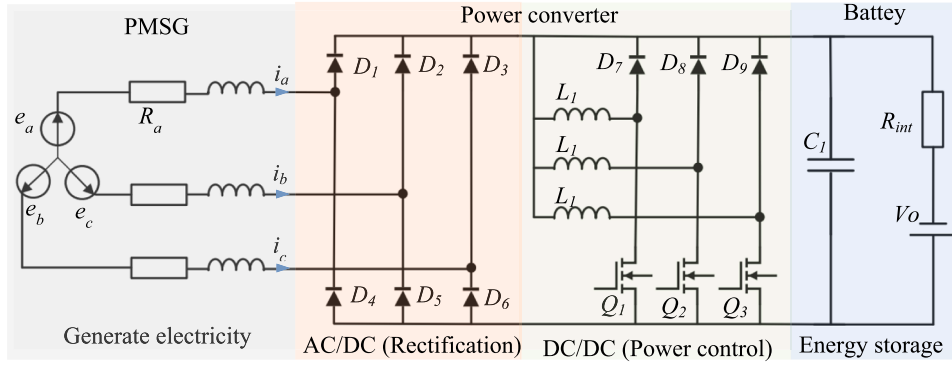


Fig. 19. Schematic of energy harvesting.

response to comments 6 and 7). Combined with Eq. (19), we can obtain the expression of the equivalent electromagnetic damping $c_{PMSG,q} = \frac{3k_e^2}{6R_a + 4R_{L,q}}, (q = r, l)$.

As is well known, the equivalent electromagnetic damping of the PMSG is directly related to the measured output power. In summary, Eq. (19) establishes a crucial connection between the PMSG's electromagnetic parameters and the PTO output power. More specifically, this implies that we can use the actual electrical parameters of the PMSG in the PTO to calculate the electrical energy generated by the proposed WEC.

The rectifier (AC/DC) consists of passive circuits and has no impact on the equivalent load resistance $R_{L,q}$ of the PMSG. While the DC/DC converter includes switching element (MOSFET), which can adjust the equivalent load resistance $R_{L,q}$ by changing the duty cycle of MOSFET [66,67]. According to the circuit impedance matching principle, the relationship between the equivalent load resistance and duty cycle can be expressed as follows:

$$R_{L,q} = \left(\frac{1 - D_q}{D_q} \right)^2 R_{int}, (q = r, l) \quad (20)$$

where D_q denote the duty cycle of the DC/DC converter in the corresponding PTO.

Furthermore, Dividing Eq. (19) by the angular velocity $\dot{\theta}_{g,q}$ of the PMSG gives the equivalent electromagnetic damping $c_{PMSG,q}$, and then substituting Eq. (20) into this result yields the equivalent electromagnetic damping for each PMSG.

$$c_{PMSG,q} = \frac{3k_e^2}{6R_a + 4\left(\frac{1-D_q}{D_q}\right)^2 R_{int}}, (q = r, l) \quad (21)$$

Based on the above analysis, we know that Eq. (21) is derived from the validated formulas Eq. (19) and Eq. (20) in the literature through rigorous mathematical derivation, providing strong evidence for its validity.

The MDD-WEC includes two types of PTOs: translation and rotation. When $q = r$, this refers to the translation PTO that utilizes a ball screw transmission structure. The angular velocity of the PMSG in the translation PTO can be obtained based on the ball screw lead [49] and is expressed as $\dot{\theta}_{g,r} = \frac{2\pi}{l_{lead}} \dot{x}_{pto,r}$.

When $q = l$, this refers to the rotation PTO that uses an epicyclic gearbox transmission structure. The angular velocity of the PMSG in the rotation PTO can be obtained based on the gearbox's transmission ratio α [68,69] and is expressed as $\dot{\theta}_{g,l} = \dot{\theta}_{pto,l} + \alpha \dot{\theta}_{pto,l}$.

Combining these explanations and references, the mechanical angular velocity $\dot{\theta}_{g,q}$ can be expressed as:

$$\dot{\theta}_{g,q} = \begin{cases} \frac{2\pi}{l_{lead}} \dot{x}_{pto,r}, (q = r) \\ \dot{\theta}_{pto,l} + \alpha \dot{\theta}_{pto,l}, (q = l) \end{cases} \quad (22)$$

For each PTO, its instantaneous output power can be obtained by multiplying the equivalent electromagnetic damping $c_{PMSG,q}$ by the square of the angular velocity $\dot{\theta}_{g,q}^2$ of the corresponding PMSG, i.e., $P_{ins,q} = c_{PMSG,q} \dot{\theta}_{g,q}^2, (q = r, l)$.

Given that the proposed WEC has three translation PTOs ($r = 1, 2, 3$) and two rotation PTOs ($l = 4, 5$), the instantaneous power of all five PTOs can be summed as $P_{allins} = \sum_{r=1}^3 c_{PMSG,r} \dot{\theta}_{g,r}^2 + \sum_{l=4}^5 c_{PMSG,l} \dot{\theta}_{g,l}^2$.

Furthermore, according to the work-energy theorem and the concept of integral [48,49], the average output power $\bar{P}_{all}$ generated by the proposed WEC during a specific motion period can be calculated by

$$\bar{P}_{all} = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} \left[\sum_{r=1}^3 c_{PMSG,r} \left(\frac{2\pi}{l_{lead}} \dot{x}_{pto,r} \right)^2 + \sum_{l=4}^5 c_{PMSG,l} \left(\dot{\theta}_{pto,l} + \alpha \dot{\theta}_{pto,l} \right)^2 \right] dt \quad (23)$$

where t_0 and t_f denote the start and end times of the system operation, respectively.

Noted that, Eqs. (21) and (22) are key terms in the PTO force formulations. Moreover, Eq. (23) describes the total average output power of the proposed WEC based on $c_{PMSG,q}$ and $\dot{\theta}_{g,q}$ in the WEC's dynamic equation. Therefore, Eqs. (21) to (23) are closely related to the WEC's dynamic equations.

The power per unit width of the incident wave can be expressed as follows for both regular and irregular waves:

$$P_{wave} = \begin{cases} \frac{\rho g H^2 \omega}{16 k_d} \left(1 + \frac{2 k_d(\omega) D_e}{\sinh(2 k_d(\omega) D_e)} \right) \dots \text{Regular wave} \\ \frac{1}{2} \rho g \int_0^\infty S(\omega) \frac{\omega}{k_d(\omega)} \left(1 + \frac{2 k_d(\omega) D_e}{\sinh(2 k_d(\omega) D_e)} \right) d\omega \dots \text{Irregular wave} \end{cases} \quad (24)$$

where the D_e and k_d represent the water depth and wave number, respectively.

An important index commonly used to evaluate the energy harvesting performance of a WEC is the capture width ratio (CWR) [70], which can be expressed as

$$CWR = \frac{\bar{P}_{all}}{P_{wave} W_D}$$

where W_D is the physics width of the MDD-WEC.

4. Experimental validation of mathematical model

In order to validate the accuracy of the mathematical model, the

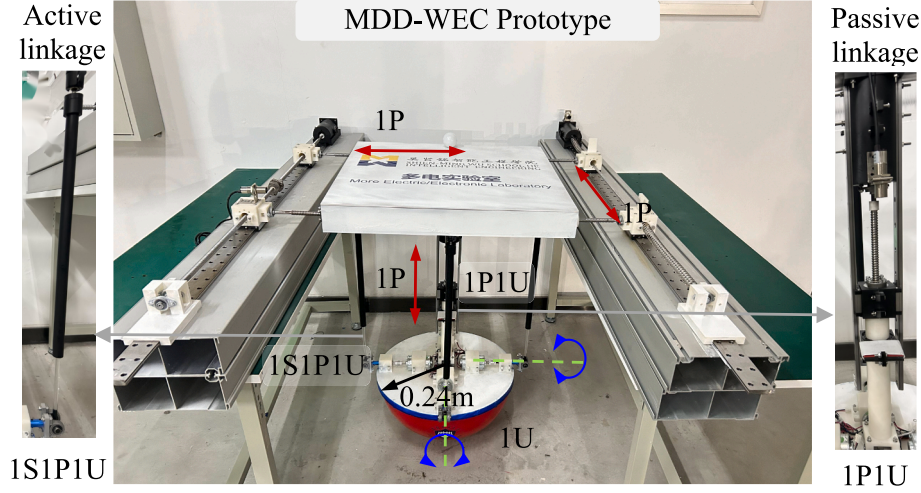


Fig. 20. The reduced scale prototype of MDD-WEC.

reduced scale prototype of the MDD-WEC is manufactured, and the wave tank experimental is conducted to test the floating body's motion displacement under irregular waves while considering the effects of multiple PTOs.

4.1. Experimental setup

Based on the structural design in Section 2, the MDD-WEC prototype is constructed on a 1:10 scale according to the Froude similarity criterion. As shown in Fig. 20, the floating body radius of the reduced-scale MDD-WEC prototype is 0.24 m. With a structural configuration that includes active and passive kinematic linkages, the floating body can achieve two-dimensional rotation (pitch and surge) and three-dimensional translation (surge, sway, and heave) within a specific range.

The MDD-WEC reduced scale prototype is tested at the wave tank operated by the South China University of Technology. As shown in Fig. 21, the basin measures 14 m $\times$ 1 m and is 0.9 m deep, with water added to reach a maximum depth of 1.7 m. The wave tank can generate both regular and irregular waves. The main structure parameters of wave tank are presented in Table 2.

As shown in Fig. 22, the experimental prototype is fixed above the wave tank using an F-shaped fixture. The floating body is designed to

Table 2

The main structure parameters of the experimental tank.

Wave height range	Wave frequencies range	Depth range in front of board	Wave duration	Control interval
0.02 ~ 0.3 m	2.09 ~ 12.57 rad/s	0.2 ~ 0.8 m	30 ~ 1800 s	0.02 s

open, enabling the placement of gravitational blocks and a MEMS inertial sensor, ensuring the required mass distribution and allowing for the measurement of the floating body's rotation angle in the pitch and roll directions. Motion displacement in the heave, surge, and sway directions is measured using three laser rangefinders. When waves flow through the MDD-WEC, the floating body begins to oscillate with the waves. Using the above sensors, the floating body's motion displacement and rotation angles are recorded and transferred to the data processing system via Bluetooth.

The PTO structural parameters used for the test are provided in Table 3. Moreover, the structurally decoupled design ensures that the operating axes of the PTOs in the WEC align one-to-one with the motion degrees of freedom (DOFs) of the floating body. This design enables each PTO to operate independently, meaning that the motion of one PTO does

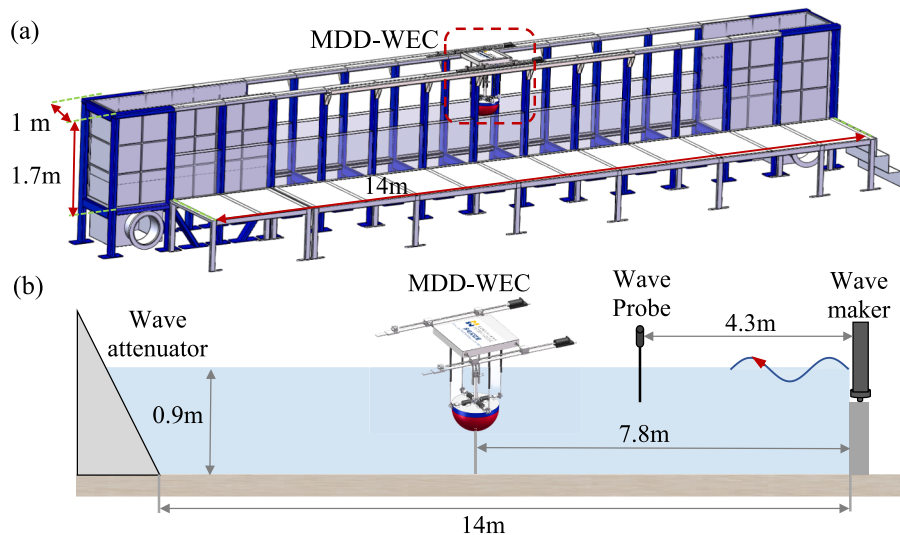


Fig. 21. The layout and dimensions for MDD-WEC prototype tests: (a) 3-D layout, (b) dimension and position placement.

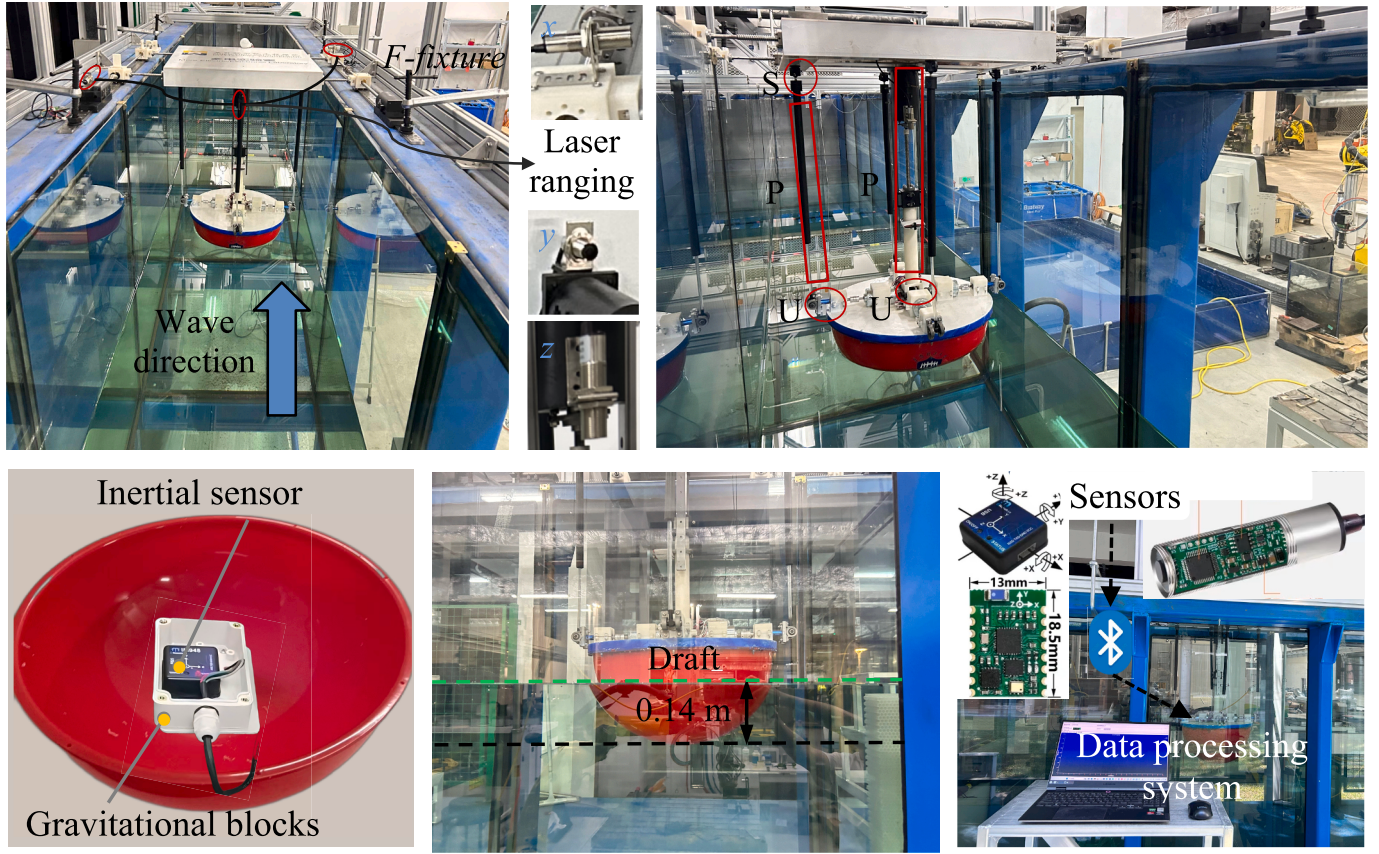


Fig. 22. Experimental setup for scaled prototype testing of the MDD-WEC.

Table 3

PTO parameters of the reduced scale MDD-WEC in wave tank experiments.

Symbol	Parameter	Value	Unit
I_S	Rotational inertia of ball screw	2.1×10^{-3}	kg·m ²
I_P	Rotational inertia of PMSG rotor	3×10^{-3}	kg·m ²
I_C	Rotational inertia of coupler	3.5×10^{-5}	kg·m ²
I_G	Rotational inertia of gearbox	1.36×10^{-4}	kg·m ²
R_a	Stator phase resistance	2.62×10^{-3}	Ohm
k_e	Voltage constant	9.819	V _{peak} L-L/(rad/s)
L	Inductance	1.87×10^{-4}	mH
R_{int}	Internal resistance of battery	5.5×10^{-4}	Ohm
l_{lead}	Lead of ball screw	0.04	m
λ	Translation PTO's transmission efficiency	0.97	/
η	Rotation PTO's transmission efficiency	0.97	/

not affect the others. Consequently, we can assign individual stiffness and damping coefficients to each PTO. Specifically, the PTO stiffness includes $k_{T,1} = 150\text{N/m}$ (surge), $k_{T,2} = 150\text{N/m}$ (sway), $k_{T,3} = 0\text{N/m}$ (heave), $k_{S,4} = 0\text{N/m}$ (roll), $k_{S,5} = 0\text{N/m}$ (pitch), while the PTO electromagnetic damping coefficients are $c_{PMSG,1} = 16\text{Ns/m}$ (surge), $c_{PMSG,2} = 16\text{Ns/m}$ (sway), $c_{PMSG,3} = 14.2\text{Ns/m}$ (heave), $c_{PMSG,4} = 6\text{Nms/rad}$ (roll), and $c_{PMSG,5} = 6\text{Nms/rad}$ (pitch). Before the experiment begins, equipment measurement data must be calibrated. This includes wave height and frequency generated in the wave tank, the floating body's rotation angle measured by the MEMS inertial sensor, and its motion displacement recorded by the laser rangefinder. Calibration is repeated several times until the average error is sufficiently small to meet the

equipment's normal operating criteria.

4.2. Model verification

In this section, the floating body undergoes hydrostatic free decay tests, and its motion response is shown in Fig. 23 under various modes. The free decay tests aim to assess whether the mesh quality in ANSYS AQWA is sufficiently refined to meet the requirements for hydrodynamic numerical analysis [24]. It should be noted that each free decay test focuses on a single degree of freedom, which is consistent with some classical literature [71,72]. The setup and boundary conditions for the free decay tests in both the wave tank experimental and numerical simulation setups are as follows:

Experimental setup: During each test, all DOFs, aside from those being evaluated, are locked. For example, horizontal motions (surge/sway) and rotational motions (roll/pitch) are mechanically locked using rigid fixtures during heave decay tests. Notably, this is successfully established relying on our structural decoupled design, where multiple PTOs operate independently, so restricting other PTO directions does not affect the free decay behavior in the tested direction. Boundary conditions: During free decay motion, the generators are set to a no-load condition to eliminate electromagnetic damping interference. Note that friction damping from low-friction hinges and linear guides (well-lubricated) is deemed negligible in these tests. The surge PTO and sway PTO are installed with spring stiffness coefficients $k_{T,1} = 150\text{N/m}$ and $k_{T,2} = 150\text{Ns/m}$, while the other PTOs do not include spring stiffness. Additionally, initial displacements (0.2 m for heave, 0.2 m for surge, and 0.1 rad for pitch) are manually applied under calm water conditions and then released to initiate free decay. Finally, the WEC's free decay amplitude is acquired using calibrated sensors.

Numerical simulation setup: The free decay simulation is performed in ANSYS AQWA, with the exception of the direction being tested, where

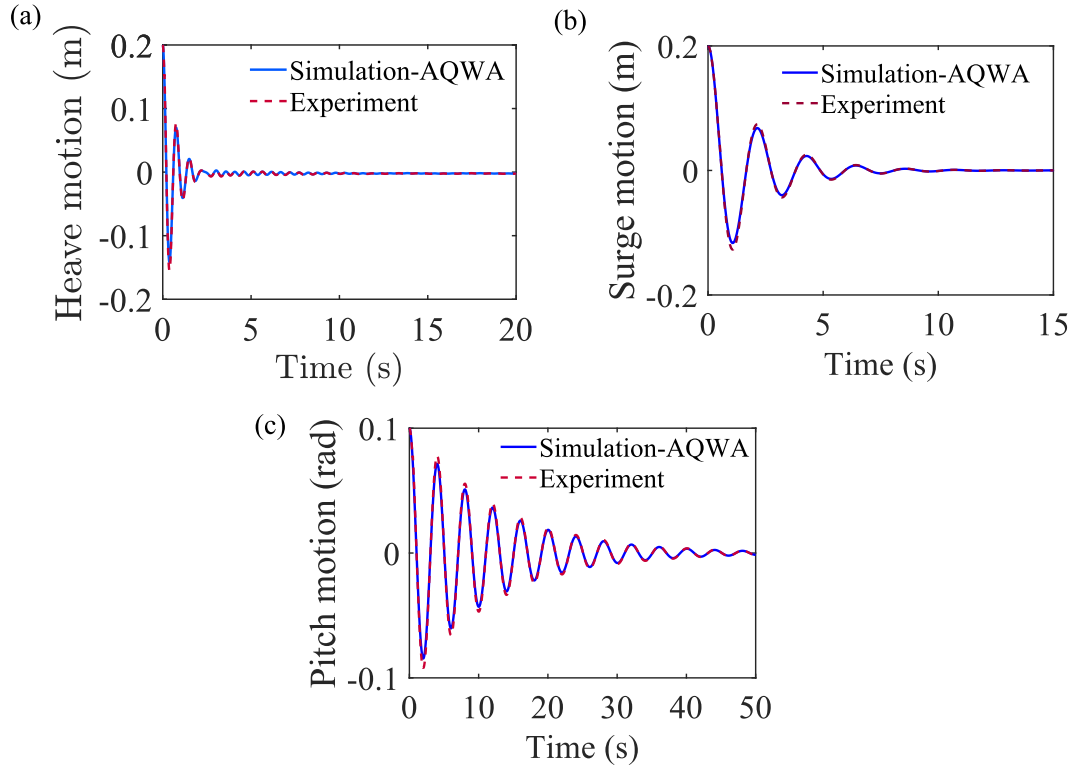


Fig. 23. Hydrostatic free decay tests of the MDDWEC: (a) under heave mode, (b) under surge mode, and (c) under pitch mode.

the WEC's displacement in all other directions is set to 0. This setup is conceptually the same as the experimental test in the wave tank. Boundary conditions: Apart from the stiffness settings for $k_{T,1} = 150\text{N/m}$ (surge) and $k_{T,2} = 150\text{N/m}$ (sway), the stiffness and damping coefficients for the other PTOs are set to 0. Initial displacements (0.2 m for heave, 0.2 m for surge, and 0.1 rad for pitch) are the same as those set in the wave tank experiment. The final WEC free decay amplitude is obtained using the ANSYS AQWA solver.

One can see that the experimental results agree well with those of the numerical simulation. These results effectively confirm the accuracy of the hydrodynamic model and also indicate that the mesh quality is sufficiently high to meet the requirements.

Furthermore, following the Froude similarity criterion, wave tank experiment tests are performed in the two irregular wave conditions, with significant wave heights and corresponding peak frequency characteristics defined as $H_{1/3} = 0.15\text{ m}$, $\omega_p = 3.31\text{ rad/s}$, and $H_{1/3} = 0.23\text{ m}$, $\omega_p = 2.61\text{ rad/s}$. The duration of each run is 80 s. The corresponding wave spectrum density function $S(\omega) = 18.63 \frac{\omega_p^4 H_{1/3}^2}{\pi^4 \omega^5} \exp\left(-\frac{\omega_p^4 71.48}{\pi^4 \omega^4}\right)$ is based on JONSWAP spectrum for the target sea region.

To ensure the credibility of the experimental results, statistical uncertainty calculations were performed. Five repetitions of the two wave conditions are conducted to determine the average motion amplitudes for heave, surge, and pitch. Table 4 details the relative errors for each measurement compared to the five-test average, along with their uncertainties. The uncertainties corresponding to different operation modes are defined as the standard deviation of motion response amplitudes divided by the square root of the number of measurements. Results indicate that, under these wave conditions, the MDD-WEC's motion amplitudes show relative errors below 5 %. These findings validate the reliability of the experimental tests.

Fig. 24 presents a comparison between the numerical calculation and wave tank test results for heave, surge, and pitch of the floating body under different wave conditions. It should be noted that, based on the Froude similarity criterion (1:10), the experimental results correspond to one-tenth of the full-scale design parameters. From Fig. 16, while the floating body's motion responses are initially smaller in the experiment than in the numerical simulation, as time goes on, the experimental and

Table 4

Measurement uncertainties and relative errors for motion responses across various modes.

Conditions		Heave (m)	Error	Uncertainty (m)	Surge (m)	Error	Uncertainty (m)	Pitch (rad)	Error	Uncertainty (m)
C1	1	0.032	0.6 %	4.1×10^{-4}	0.044	4.3 %	5.9×10^{-4}	0.0118	2.2 %	1.57×10^{-4}
	2	0.033	3.8 %		0.043	1.9 %		0.0105	4.2 %	
	3	0.031	2.5 %		0.041	2.8 %		0.0121	1.5 %	
	4	0.032	0.6 %		0.042	0.5 %		0.0109	0.6 %	
	5	0.031	2.5 %		0.041	2.8 %		0.0119	4.0 %	
C2	1	0.065	1.3 %	9.8×10^{-4}	0.132	1.6 %	1.2×10^{-3}	0.0225	0.71 %	3.50×10^{-4}
	2	0.067	4.3 %		0.135	0.6 %		0.0232	2.4 %	
	3	0.062	3.4 %		0.130	3.1 %		0.0237	4.6 %	
	4	0.064	0.3 %		0.138	2.8 %		0.0218	3.8 %	
	5	0.063	1.9 %		0.136	1.3 %		0.0221	2.5 %	

Note: C1 and C2 represent the two irregular wave conditions, respectively. Where, C1: $H_{1/3} = 0.15\text{ m}$, $\omega_p = 3.31\text{ rad/s}$; C2: $H_{1/3} = 0.23\text{ m}$, $\omega_p = 2.61\text{ rad/s}$.

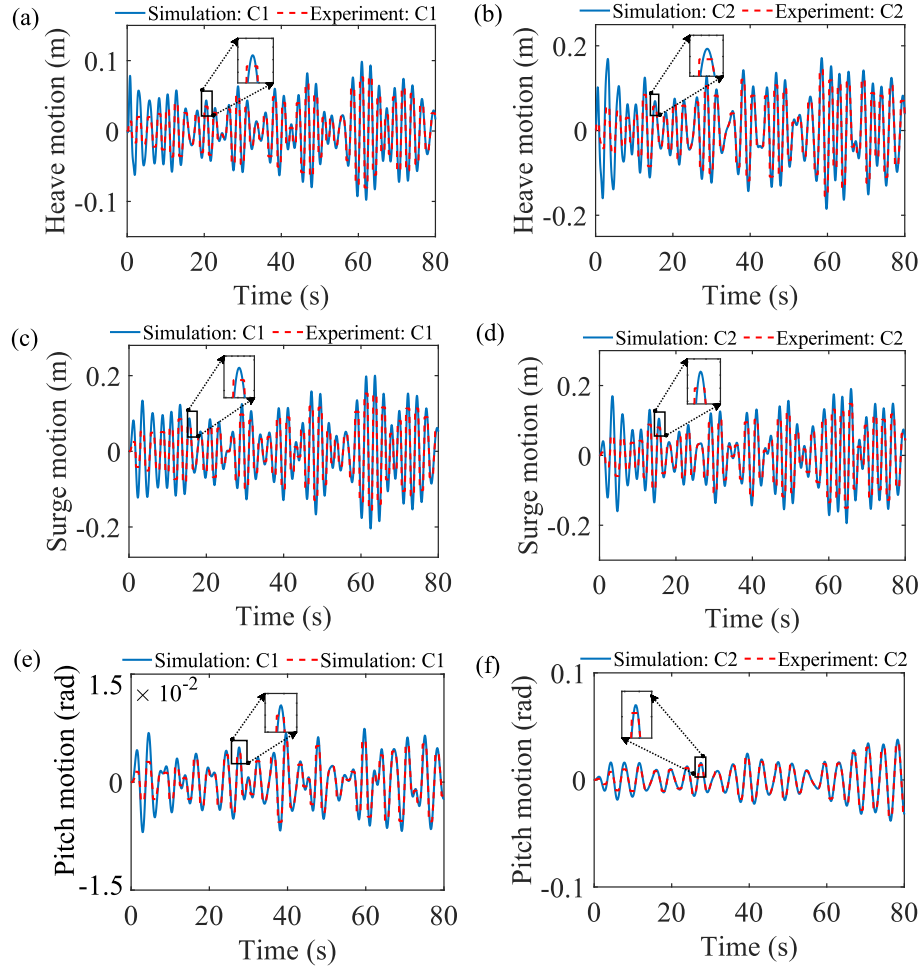


Fig. 24. Comparison of experimental and numerical motion responses of the MDD-WEC: (a) heave motion under wave condition C1, (b) heave motion under wave condition C2, (c) surge motion under wave condition C1, (d) surge motion under wave condition C2, (e) pitch motion under wave condition C1 and (f) pitch motion under wave condition C2.

simulated results show strong agreement in both amplitude and phase. This initial difference arises because the experimental data for the MDD-WEC were recorded from the start of wave generation. Thus, this agreement validates the reliability of both the mathematical model and the calculation program used. Additionally, the heave, surge, and pitch curves stop at their highest and lowest points. This phenomenon occurs because the floating body needs a certain counteracting force to move in the opposite direction due to damping effects.

In addition to the time series comparison, a frequency-domain analysis comparing the spectral characteristics of the motion response is also performed using the data of simulations and experiments. The Welch method [73] is employed to analyze the Power Spectral Densities (PSDs) of numerical and experimental time series for heave, pitch, and surge motions. As shown in Fig. 25, the dominant peaks in the motion PSDs align closely between simulations and experiments, indicating that the model accurately captures the primary dynamic characteristics of the system. Although some discrepancies exist at certain non-dominant frequencies, they are not critical to the system's behaviour [74,75]. Combined with the strong temporal consistency observed in the time series (see Fig. 24), this analysis confirms the validity of the mathematical model.

5. Energy harvesting performance analysis

Based on the above-validated mathematical model, this section aims to evaluate the energy harvesting performance of the full-scale MDD-

WEC under real wave conditions of the target sea region. Specifically, the multiple PTOs parameter analysis is first performed to reveal the effect of different PTO configurations on the energy harvesting performance of the system under regular and irregular wave conditions (See Fig. 26). To further assess its potential, we compare it with the widely applied point absorber WEC (PAWEC) [20] and the PCWEC [33] which serves as a benchmark for MDWEC. Two comparison scenarios are considered: prevailing local wave conditions and varying wave heights and frequencies. These scenarios can well cover the wave condition during the actual operation process of the WEC. Note that, based on the reduced-scale prototype parameters in Table 3, the PTO structure parameters of the full-scale MDD-WEC can be obtained using the Froude similarity criterion and will not be presented here for brevity. For a fair comparison, the PTO configuration and structural parameters of the PAWEC and PCWEC are designed to align with those of the MDD-WEC.

5.1. Multiple PTOs parameter analysis

In this case, the impact of the stiffness and electromagnetic damping coefficients of the multi-PTO on the energy harvesting performance of the MDD-WEC is analyzed through numerical simulations. The analysis is conducted under prevailing wave conditions with a significant wave height of 1.5 m, a wave frequency of 1.05 rad/s (as detailed in Section 2.1), and a wave direction of 110 degree.

Consider first the harvested power under regular wave conditions in Fig. 27. It can be seen that by adjusting the PTO stiffness and

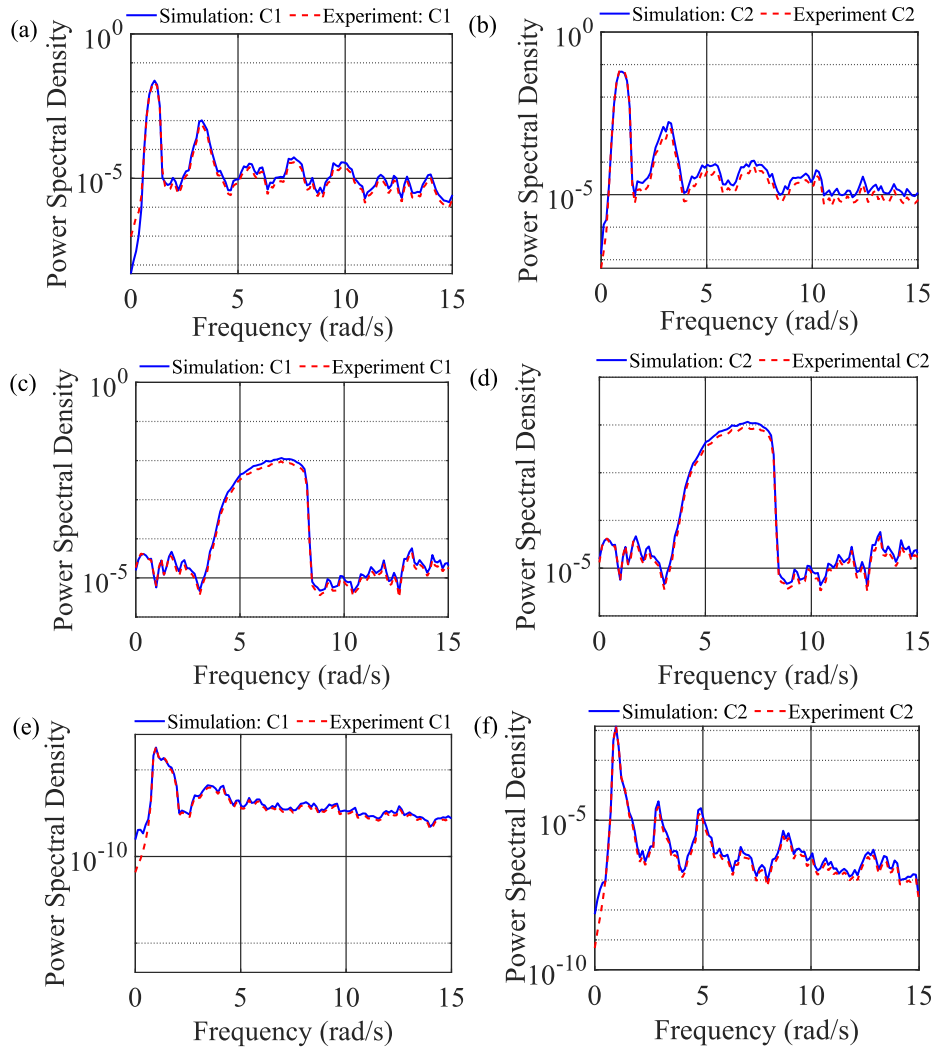


Fig. 25. Power spectral densities comparison of experimental and numerical motion responses of the proposed WEC: (a) heave motion under wave condition C1, (b) heave motion under wave condition C2, (c) surge motion under wave condition C1, (d) surge motion under wave condition C2, (e) pitch motion under wave condition C1 and (f) pitch motion under wave condition C2.

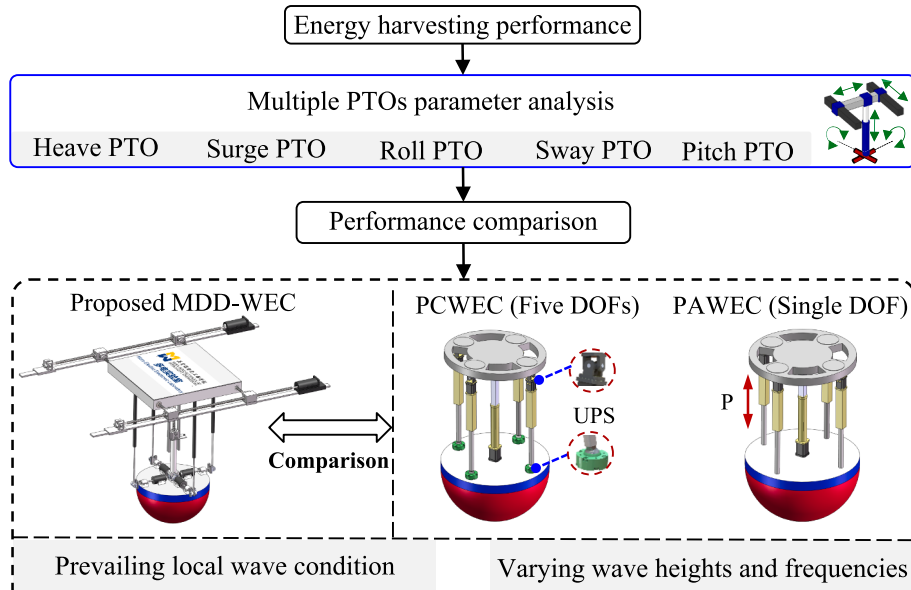


Fig. 26. Framework of investigation and comparison of MDD-WEC.

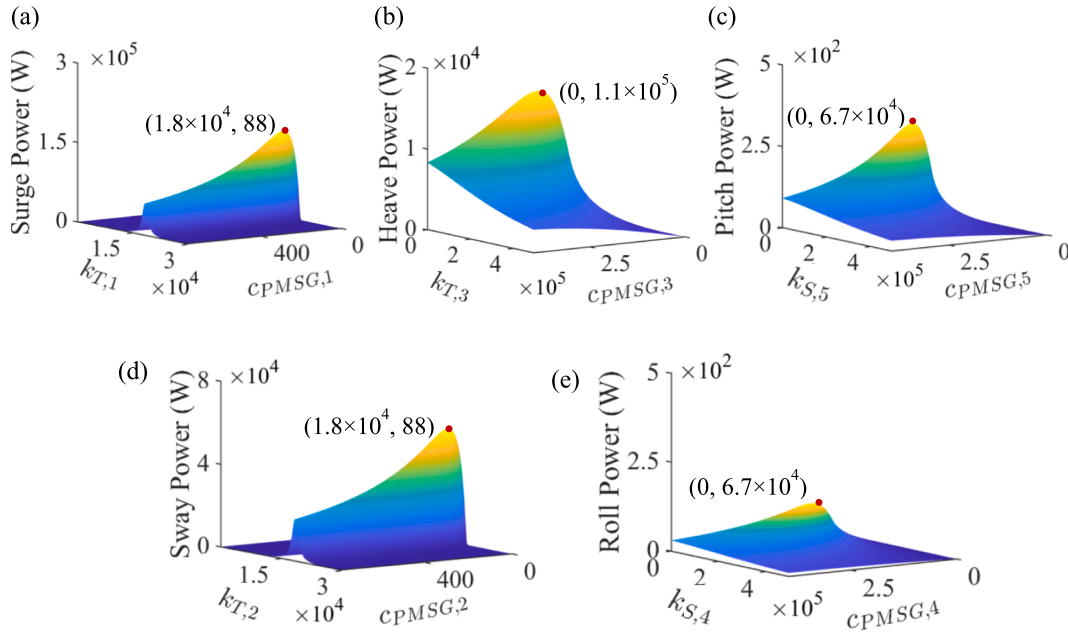


Fig. 27. Effect of the stiffness and electromagnetic damping coefficients of multi-PTO on the output power of the MDD-WEC under regular wave condition: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, and (e) roll PTO.

electromagnetic damping coefficients, the output power of the WEC experiences significant fluctuations. The stiffness and electromagnetic damping coefficients that maximize the system's output power can be regarded as the optimal parameters for the PTOs under the given wave conditions, as marked by a red dot in Fig. 27. Based on these points, we find that, due to the symmetry of the proposed structure, the optimal parameters for the surge PTO and pitch PTO are the same as those for the sway PTO and roll PTO, respectively. Interestingly, although the wave frequency and wave height are constant, the optimal parameters for the heave PTO, surge PTO, and pitch PTO are not the same. This is because the MDD-WEC has different resonance frequencies due to distinct hydrodynamic effects (hydrostatic stiffness and radiation damping) in the heave, surge, and pitch directions. Thus, to obtain maximum output

power, the optimal parameters for the corresponding PTOs inevitably differ.

Another notable observation in Fig. 28 is that, while the MDD-WEC has multiple PTOs, each PTO parameter only affects its own output power, without impacting the output power of other PTOs. This can be attributed to the decoupled design, which ensures that the operating axes of each PTO align one-to-one with the motion DOFs of the floating body, thereby eliminating the interaction between multi-PTOs. Moreover, this independent operation characteristic means that each PTO in the MDD-WEC can independently optimize its performance to absorb more wave energy according to wave conditions.

To further investigate the energy harvesting performance of the MDD-WEC, irregular wave with significant wave height of 1.5 m and

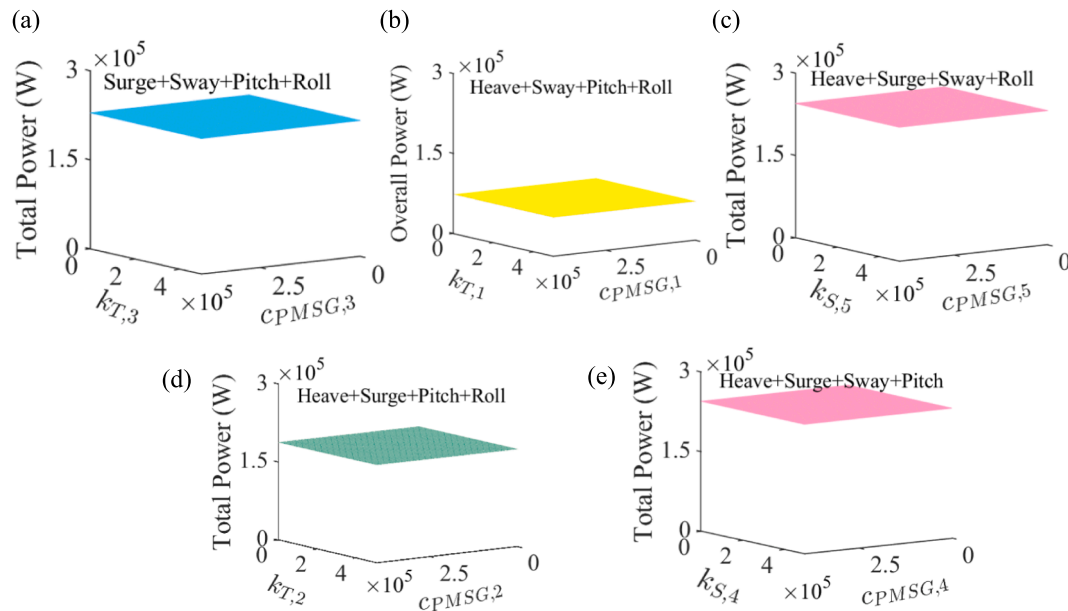


Fig. 28. Effect of the stiffness and electromagnetic damping coefficients of a single PTO on the total output power of other PTOs in the MDD-WEC under regular wave conditions: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, and (e) roll PTO.

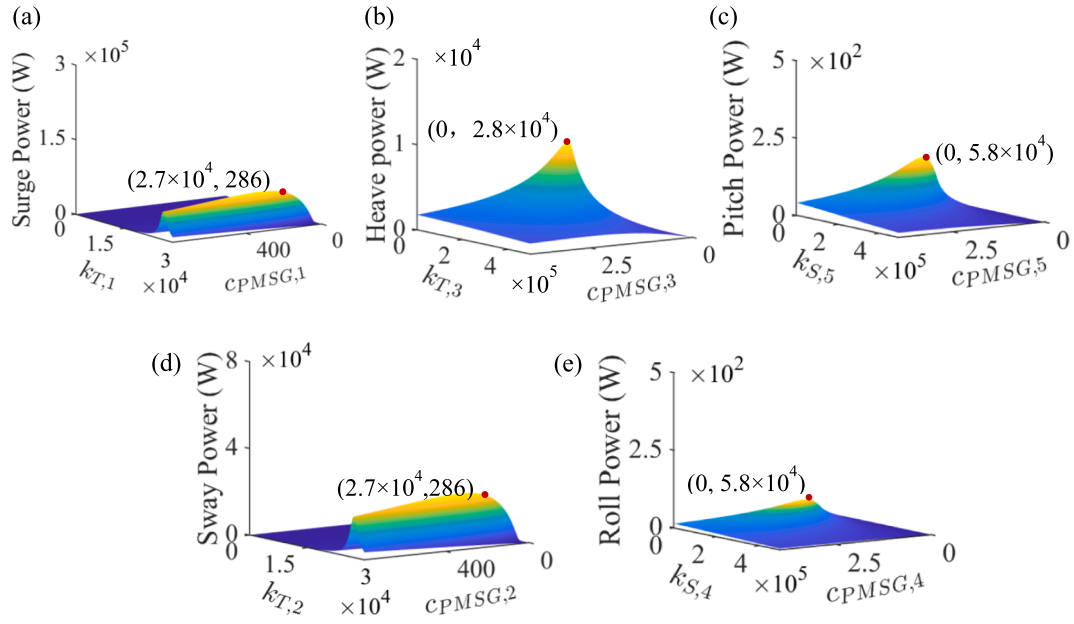


Fig. 29. Effect of the stiffness and electromagnetic damping coefficients of multi-PTO on the output power of the MDD-WEC under irregular wave condition: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, and (e) roll PTO.

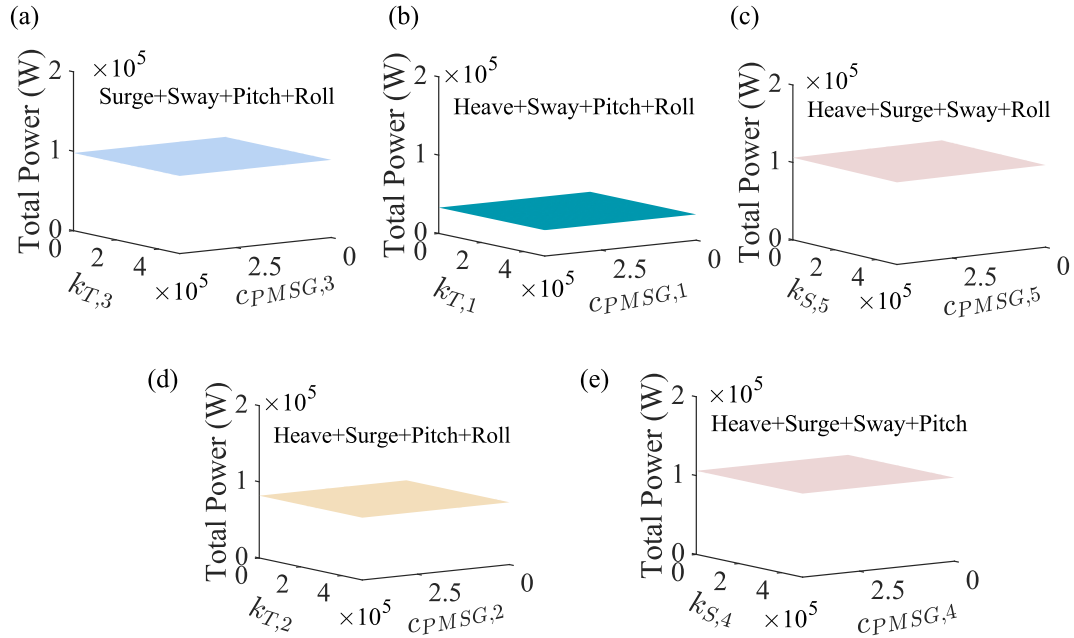


Fig. 30. Effect of the stiffness and electromagnetic damping coefficients of a single PTO on the total output power of other PTOs in the MDD-WEC under regular wave conditions: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, and (e) roll PTO.

peak frequency of 1.05 rad/s are considered to conduct test. As shown in Fig. 29, the effects of multi-PTO parameters on the MDD-WEC's output power exhibit a similar trend to those observed under regular wave conditions. Moreover, due to the decoupled design, the multiple PTOs also exhibit independent operational characteristics under irregular wave conditions (see Fig. 30). However, compared to regular waves, the optimal PTO parameters vary under irregular waves, and the maximum output power is also lower in irregular wave conditions.

5.2. Comparison under prevailing wave condition

In this case, the prevailing wave conditions, with a wave height of

1.5 m, a wave frequency of 1.05 rad/s, and three main incident wave directions, are considered for the performance comparison. The motion equations of the PAWEC and PCWEC are referenced from Refs. [20,33] and are not repeated here. For performance comparison, the PTO coefficients in the MDD-WEC are adopted from the above optimal stiffness and electromagnetic damping. To ensure fairness, the optimal PTO coefficients for the PCWEC and PAWEC are determined using the same analysis process as the MDD-WEC in Section 5.1.

Please understand that performing time series analysis for all possible wave conditions and PTO parameter variations may be challenging. To ensure a practical yet representative analysis, we select prevailing wave conditions including both regular and irregular waves,

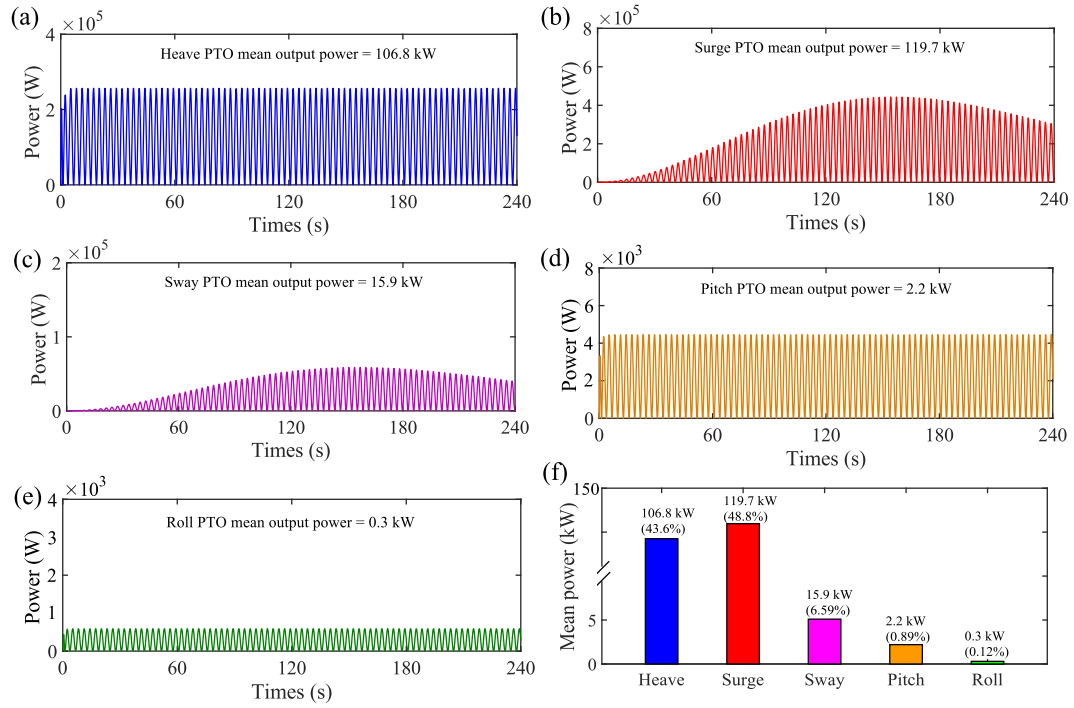


Fig. 31. Individual PTO power of the proposed WEC under regular wave condition: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, (e) roll PTO and (f) PTO mean output power distribution.

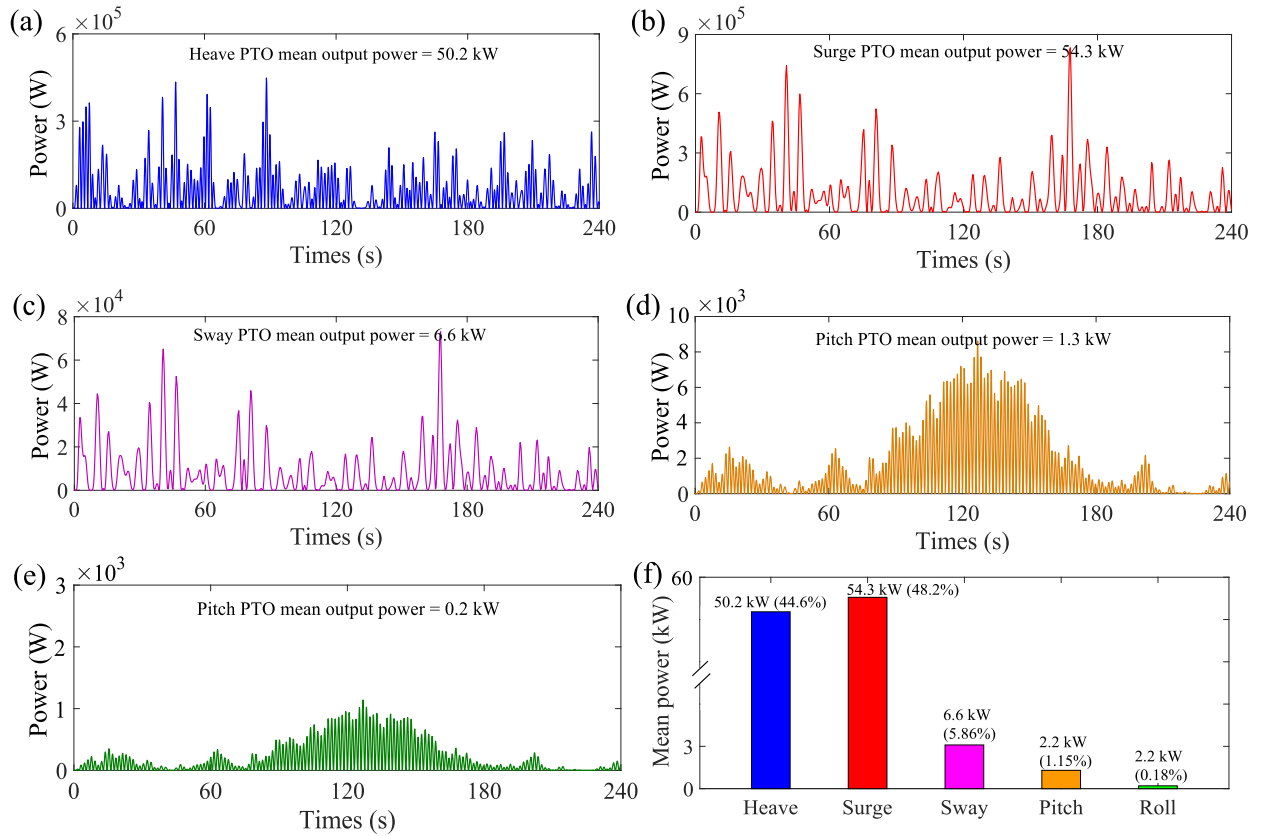


Fig. 32. Individual PTO power of the proposed WEC under irregular wave condition: (a) heave PTO, (b) surge PTO, (c) pitch PTO, (d) sway PTO, (e) roll PTO and (f) PTO mean output power distribution.

along with the corresponding optimal PTO coefficients for evaluation. As previously described (see Section 2.1), the prevailing regular wave condition is defined by a 1.5 m wave height, a 1.05 rad/s wave frequency, and a 110° primary incident wave direction. The irregular wave condition follows the JONSWAP spectrum $S(\omega) = 18.63 \frac{\omega_p^4 H_1^2}{\pi^4 \omega^5} \exp\left(-\frac{\omega_p^4 71.48}{\pi^4 \omega^4}\right)$ for our target sea region, with a 1.5 m wave height and a 1.05 rad/s peak frequency.

Fig. 31 and Fig. 32 present the individual PTO output power of the WEC under regular and irregular wave conditions, respectively. It can be observed that the surge PTO consistently exhibits the highest mean output power in both cases. Specifically, under regular wave condition, the mean output power of each PTO is 119.7 kW (surge), 106.8 kW (heave), 15.9 kW (sway), 2.2 kW (pitch), and 0.3 kW (roll), with power contributions of 48.80 % (surge), 43.60 % (heave), 6.59 % (sway), 0.89 % (roll), and 0.12 % (pitch). Under irregular wave condition, the mean output power values decrease to 54.3 kW (surge), 50.2 kW (heave), 6.6 kW (sway), 1.3 kW (pitch), and 0.2 kW (roll), with adjusted power contributions of 48.2 % (surge), 44.6 % (heave), 5.86 % (sway), 1.15 % (roll), and 0.18 % (pitch). Although the relative ranking of PTO power contributions remains consistent between regular and irregular wave conditions, the absolute mean output power of each PTO is higher under regular waves. This phenomenon occurs because irregular waves comprise multiple regular wave components with varying frequencies, and the optimal PTO parameters, calibrated for the peak wave frequency, do not ensure maximum energy harvesting across the entire frequency spectrum.

Based on above analysis, Fig. 33 further compares the instantaneous absorbed power over time for the MDD-WEC, PAWEC, and PCWEC. It shows that, regardless of whether the wave condition is irregular or regular, the MDD-WEC exhibits significant advantages in output power. Moreover, the instantaneous output power of the PAWEC and PCWEC sometimes approached zero due to the periodic nature of wave motion. In stark contrast, the instantaneous power of the MDD-WEC is not zero except at the beginning. This can be attributed to the phase differences in output power among the heave PTO, horizontal translation PTOs (surge and sway), and rotation PTOs (roll and pitch), resulting from the PTO parameter optimization based on the decoupled design. As the peaks of these PTOs are staggered, the electric power generated is more consistently delivered to energy storage. This means that power fluctuations or downtime issues are reduced in certain extent.

At an incident wave direction 110 degree, it is observed from Fig. 34

that the MDD-WEC improves the capture width ratio (CWR) by 72.5 % and 39.3 % compared to the PAWEC and PCWEC, respectively, under regular wave conditions. Even under irregular wave conditions, the increase reaches 60.8 % and 32.9 %. This is expected as the MDOF decoupling mechanism not only captures wave energy from multiple motion directions but also eliminates the energy conversion losses caused by the coupled motion in the PCWEC.

In addition, we further analysis the CWR of the above three WECs in two additional incident wave directions (See section 2.1). As shown in Fig. 34, the further the incident wave direction deviates from the floating body's horizontal axis, the lower the CWR of the PCWEC. This implies that, the orientation of the PCWEC needs to be adjusted to maintain optimal energy harvesting performance under changing wave direction. Interestingly, although the MDD-WEC also has five motion DOFs, its CWR are barely affected by the wave direction, which is similar to the single-DOF PAWEC with omnidirectional characteristics. This can be explained by the decoupled design, which aligns the PTOs' operating axes in the MDD-WEC with wave forces from different directions, effectively eliminating the negative effects of wave direction changes. This means that the proposed MDD-WEC has stable energy harvesting characteristics under changes in wave direction, and further confirms the merit of the proposed decoupled design concept.

5.3. Comparison under varying wave heights and periods

A WEC with good application potential must maintain high energy harvesting performance under varying wave conditions. As discussed in Section 2.1, although the prevailing wave conditions dominate, the wave conditions in the target sea region can vary due to external factors. To further assess the application potential of the proposed structure, this case investigates and compares the impact of varying wave heights (ranging from 0.5 to 2.3 m) and wave frequencies (ranging from 0.83 to 1.21 rad/s) on energy harvesting for the MDD-WEC, PAWEC, and PCWEC.

Consider first the output power matrices in Fig. 35 and Fig. 36. One can see that, regardless of the irregular and regular wave conditions, the MDD-WEC consistently presents significantly higher output power than the other two WECs under varying wave heights and frequencies. By further calculation, it can be observed from Fig. 35 that the MDD-WEC can increase mean output power by 858.0 % and 174.8 % compared to the PAWEC and PCWEC, respectively. In addition, under real irregular wave conditions, this increase can reach 926.1 % and 254.0 %. This indicates that the MDD-WEC has better adaptability for harvesting more wave energy under different wave conditions.

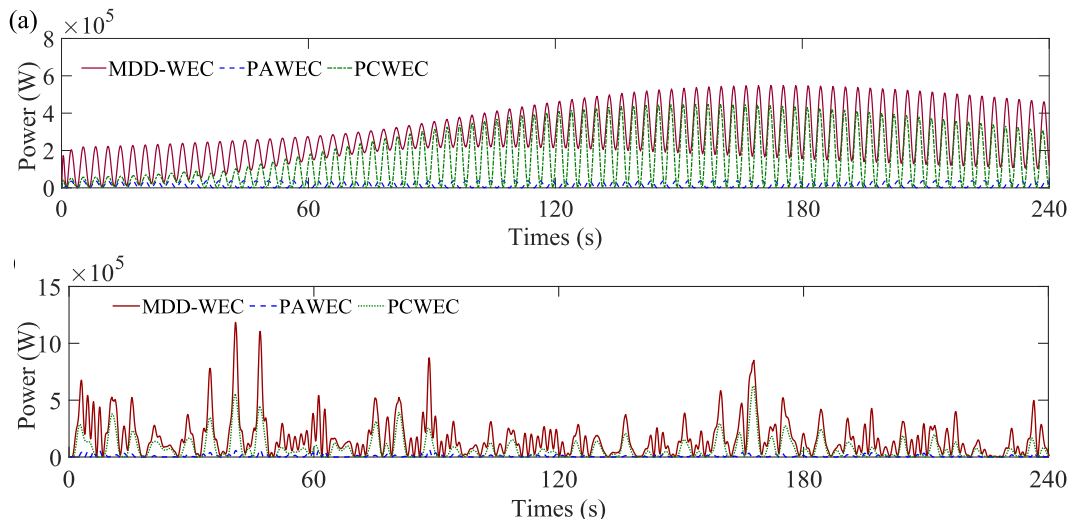


Fig. 33. Output power of the three WECs at an incident wave direction 110 degree: (a) under regular wave condition and (b) under irregular wave condition.

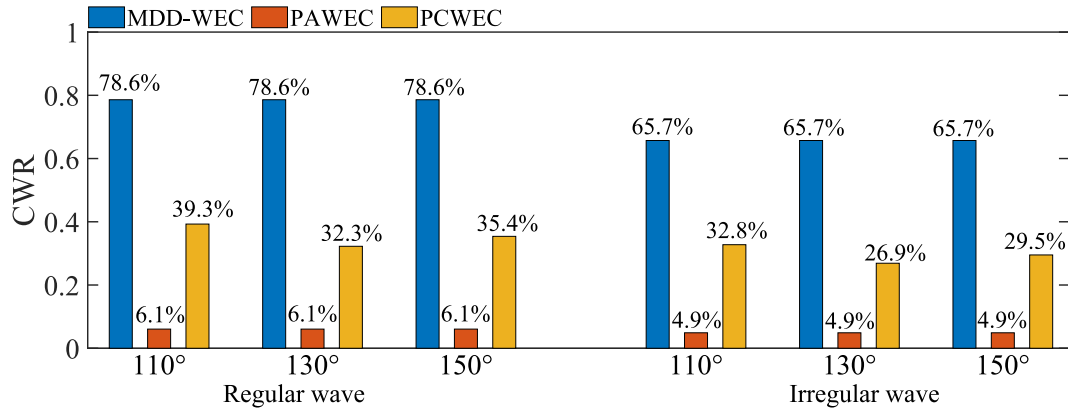


Fig. 34. Capture width ratio comparison of the three WECs under different incident wave directions.

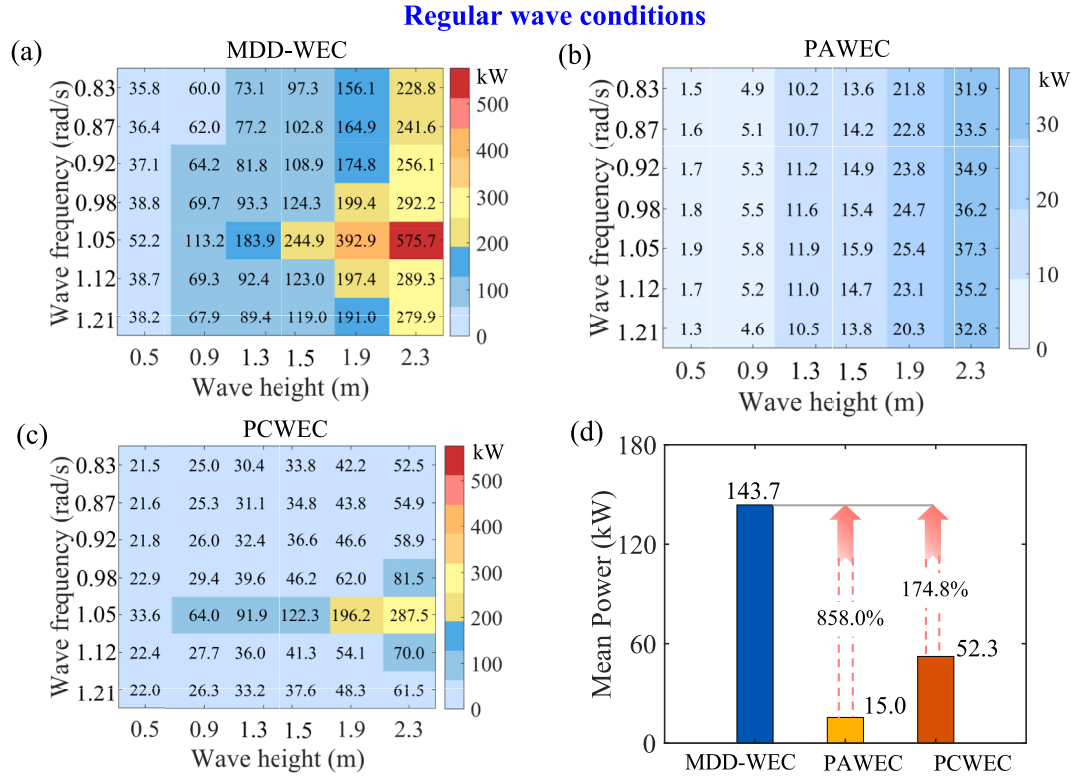


Fig. 35. Output power of the three WECs for varying wave heights and frequencies under regular wave conditions: (a) MDD-WEC, (b) PAWEC, (c) PCWEC, and (d) mean power comparison.

Using the calculated output power, the CWR of each WEC under different wave conditions is analyzed and shown in Fig. 37 and Fig. 38. When the wave frequency approaches the resonance frequency, the CWR of all three WECs increases. Interestingly, while Fig. 35 and Fig. 36 show that capture power is appreciably influenced by wave height than wave frequency, Fig. 37 and Fig. 38 reveals that wave height has barely impact on the CWR of the three WECs. This phenomenon occurs because wave frequency closer to or depart from the WEC's natural frequency significantly enhances or reduces its energy harvesting efficiency. Furthermore, the MDD-WEC achieves a significantly higher CWR than the other two WECs across varying wave heights and frequencies. Overall, these results highlight the MDD-WEC's ability to maintain high energy harvesting in varying wave conditions.

6. Conclusions

Increasing the motion DOFs of the floating body benefits the OBWCE device by improving its wave energy harvesting capacity. However, the coupled motion between the floating body and the PTO in multiple DOFs leads to a loss in energy harvesting efficiency. To address this issue, the decoupled mechanism of active drive mechanical structures is extended to the WEC's energy harvesting scenario, and a novel multi-DOF decoupled WEC (MDD-WEC) is designed and constructed. A multi-physical-domain mathematical model is developed and validated through wave tank testing, followed by multi-PTO parameter analysis and performance comparison. The following conclusions can be drawn:

- (1) Through the decoupled design, the operating axes of the PTOs align one-to-one with the floating body's motion DOFs, enabling the floating body to efficiently transfer wave energy into

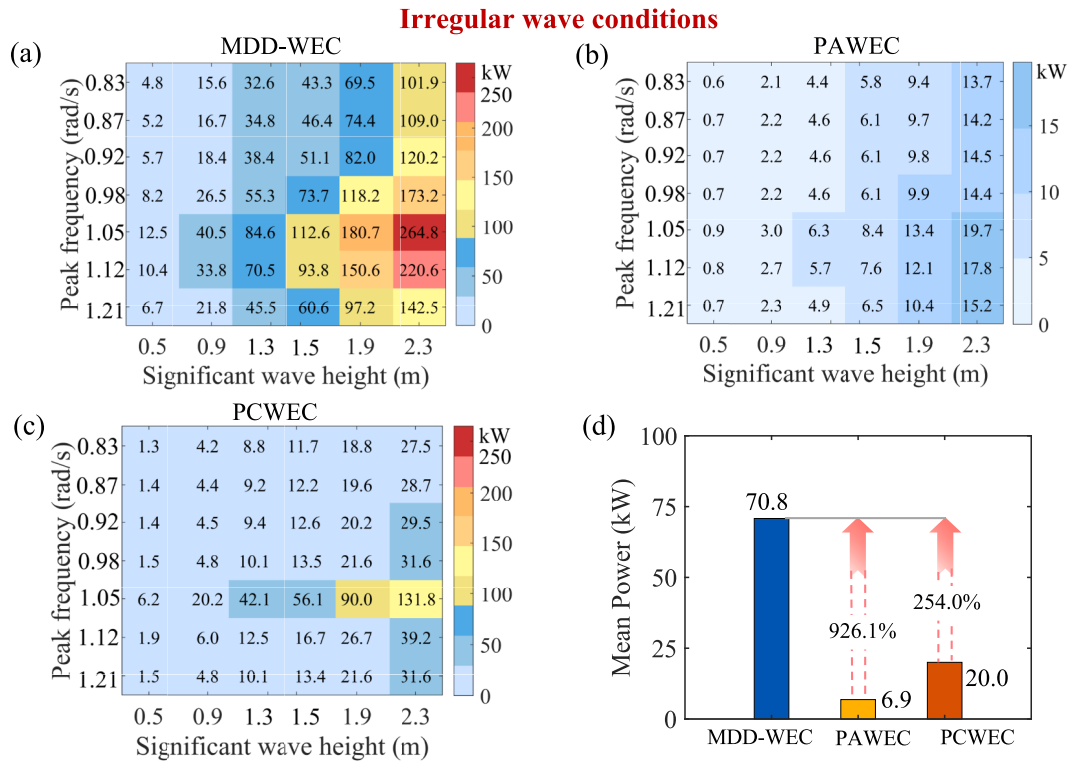


Fig. 36. Output power of the three WECs for varying wave heights and frequencies under irregular wave conditions: (a) MDD-WEC, (b) PAWEC, (c) PCWEC, and (d) mean power comparison.

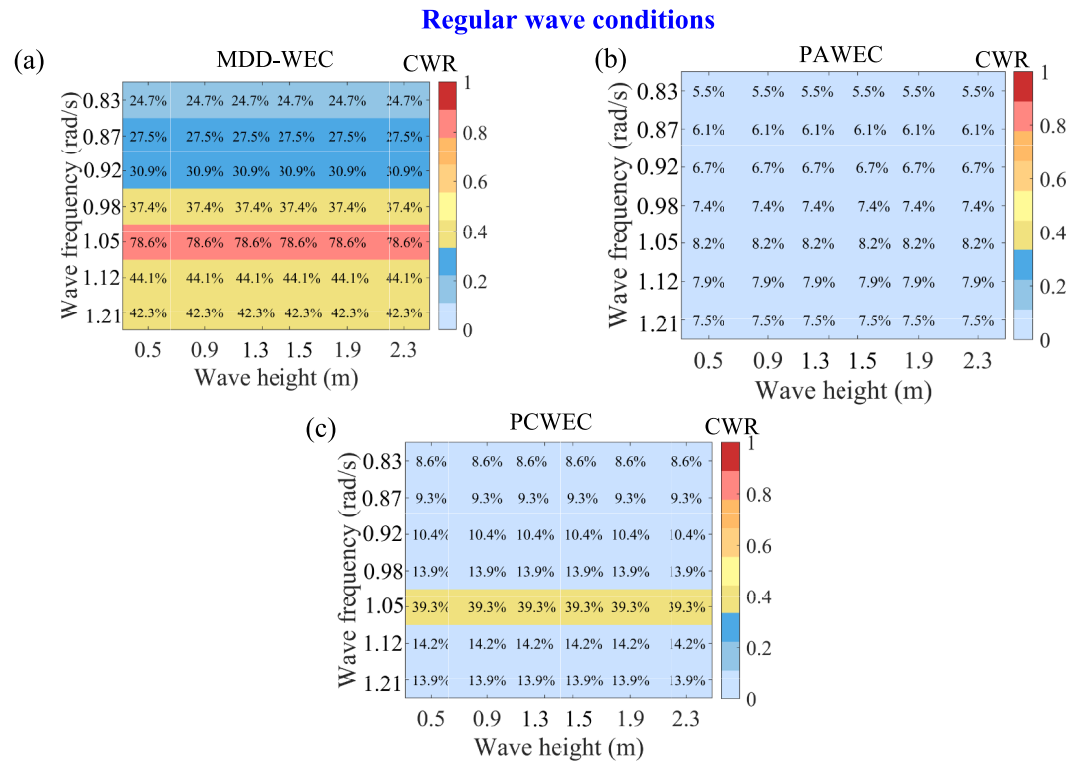


Fig. 37. Capture width ratio of the three WECs for varying wave heights and frequencies under regular wave conditions: (a) MDD-WEC, (b) PAWEC and (c) PCWEC.

electrical energy via the PTO and eliminating the energy harvesting limitations caused by coupled motion.

- (2) Regardless of the irregular and regular wave conditions, each PTO parameter only affects its own output power, without

impacting the output power of other PTOs. Benefited this characteristic, each PTO in the MDD-WEC can be independently optimized to absorb more wave energy according to wave conditions.

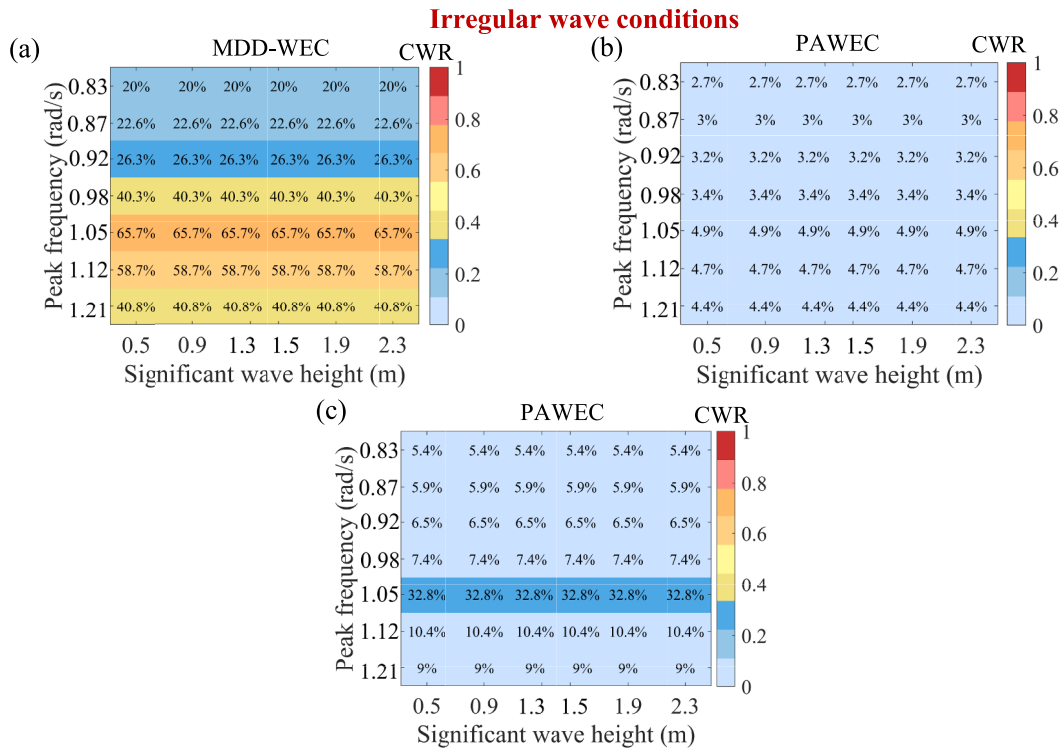


Fig. 38. Capture width ratio of the three WECs for varying wave heights and frequencies under irregular wave conditions: (a) MDD-WEC, (b) PAWEC and (c) PCWEC.

- (3) Under the prevailing regular wave conditions for the target sea region, the proposed MDD-WEC improves the capture width ratio (CWR) by 72.5 % and 39.3 % compared to the point absorber WEC (PAWEC) and the parallel configuration WEC (PCWEC) with multi-DOF, respectively. Even under irregular wave conditions, the increase reaches 60.8 % and 32.9 %, respectively.
- (4) For the proposed MDD-WEC, phase differences exist in the output power among the heave PTO, horizontal translation PTOs (surge and sway), and rotational PTOs (roll and pitch). In contrast to the PAWEC and PCWEC, the instantaneous power of the MDD-WEC is not zero except at startup, reducing power fluctuations and downtime to a certain extent.
- (5) Based on the results of parameter analysis, although the MDD-WEC has five DOFs of motion, the CWR is unaffected by the incident wave direction. Additionally, compared with the PAWEC and PCWEC, the proposed MDD-WEC exhibits a higher CWR under varying wave heights and wave frequencies.

In summary, the proposed MDD-WEC exhibits a significant performance advantage in energy harvesting under changes in external wave conditions, due to the merits of the decoupled structure. This property shows great potential for application in the engineering practice of wave energy utilization. Meanwhile, the design, modeling, experimental, and numerical analyses of the MDD-WEC also provide a useful reference for related complex MDWECs.

CRediT authorship contribution statement

Yongxing Zhang: Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Zhicong Huang:** Writing – review & editing, Supervision, Project administration, Funding acquisition. **Jing Bian:** Writing – review & editing, Resources, Project administration, Investigation. **Junwei Liu:** Supervision, Resources, Methodology. **Ning Su:** Writing – review & editing, Supervision, Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A1. Hydrodynamic parameters of MDD-WEC

The hydrodynamic parameter results for the surge, heave, and pitch modes are shown below. Notably, since the floating body is axisymmetric, the hydrodynamic parameters for surge and sway are identical, as are those for pitch and roll. Thus, the parameters for the sway and roll directions are not repeated here.

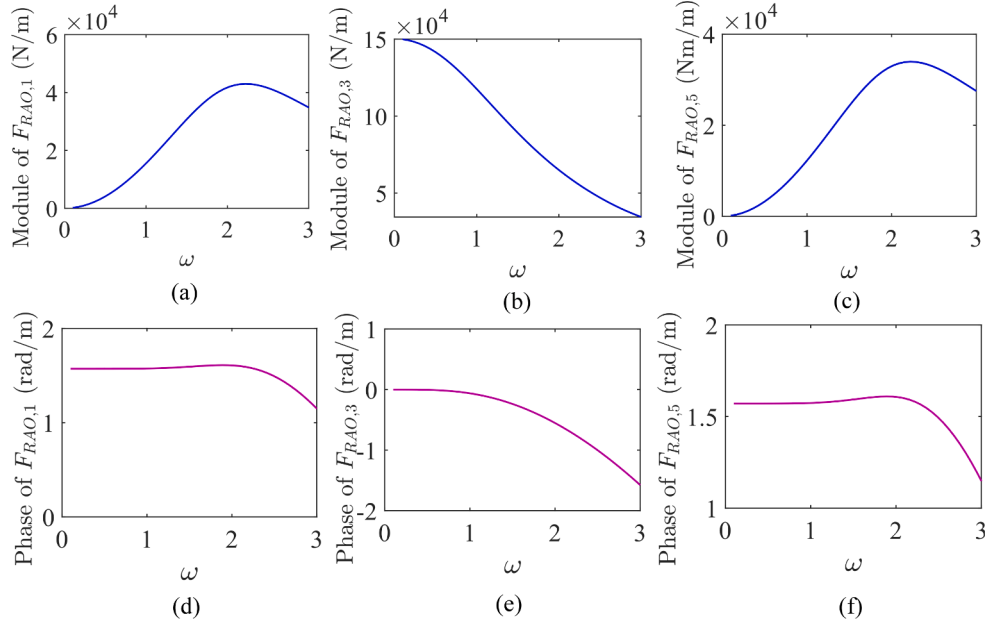


Fig. A1. The wave excitation force and moment for the MDD-WEC: (a) module of $F_{RAO,1}$, (b) module of $F_{RAO,3}$, (c) module of $F_{RAO,5}$, (d) phase of $F_{RAO,1}$, (e) phase of $F_{RAO,3}$ and (f) phase of $F_{RAO,5}$.

The original data for the infinite frequency added mass is obtained using the high-fidelity commercial software ANSYS AQWA. The comparison is performed against the numerically computed infinite frequency added mass. In this numerical process, we first solve the velocity potential using an eigenfunction expansion to obtain the added mass at finite frequencies. Then, the typical extrapolation method [35,76], based on least squares fitting, is applied to estimate the infinite frequency added mass.

As shown in Fig. A2, the original data obtained from ANSYS AQWA aligns well with the numerical calculation results. Table A1 further presents the comparison between the infinite frequency added mass obtained using ANSYS AQWA and that computed via the numerical extrapolation method. It can be observed that the deviation between the two approaches remains below 5 % across different modes, providing a reasonable level of confidence in the subsequent computation.

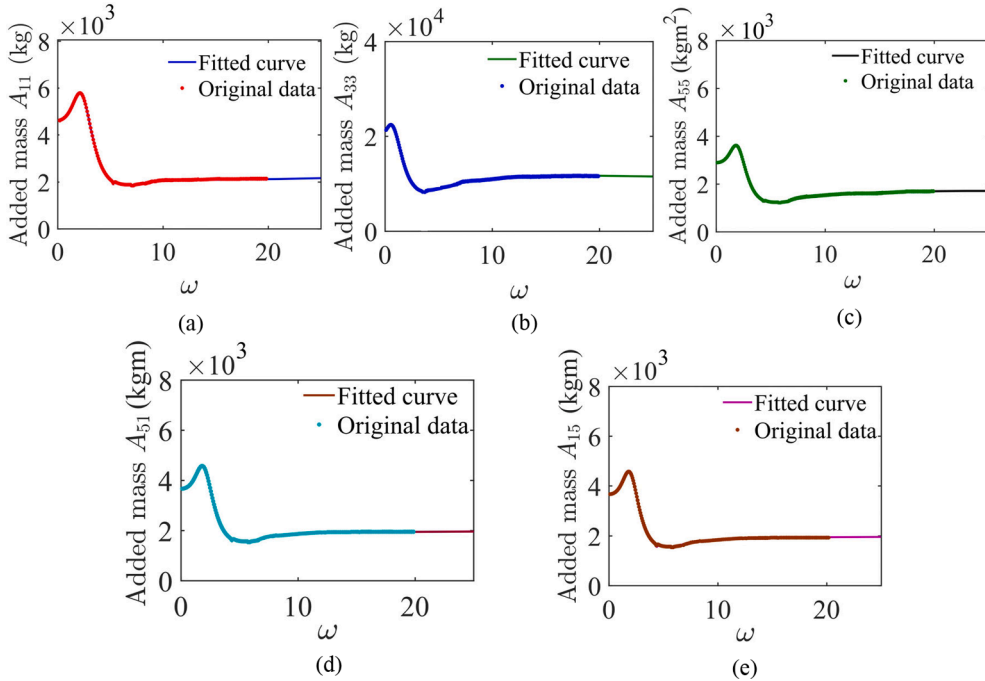


Fig. A2. Curve fitting of the infinity frequency added mass: (a) A_{11} , (b) A_{33} , (c) A_{55} , (d) A_{51} and (e) A_{15} .

Table A1
Comparison of infinite frequency added mass.

Infinite frequency added mass	Original data	Numerical calculation	Error
$A_{11}(\infty)$	2.136×10^3 kg	2.035×10^3 kg	4.73 %
$A_{33}(\infty)$	9.695×10^3 kg	9.366×10^3 kg	3.39 %
$A_{55}(\infty)$	1.276×10^3 kgm ²	1.221×10^3 kgm ²	4.31 %
$A_{51}(\infty)$	1.615×10^3 kgm	1.550×10^3 kgm	4.02 %
$A_{15}(\infty)$	1.615×10^3 kgm	1.550×10^3 kgm	4.02 %

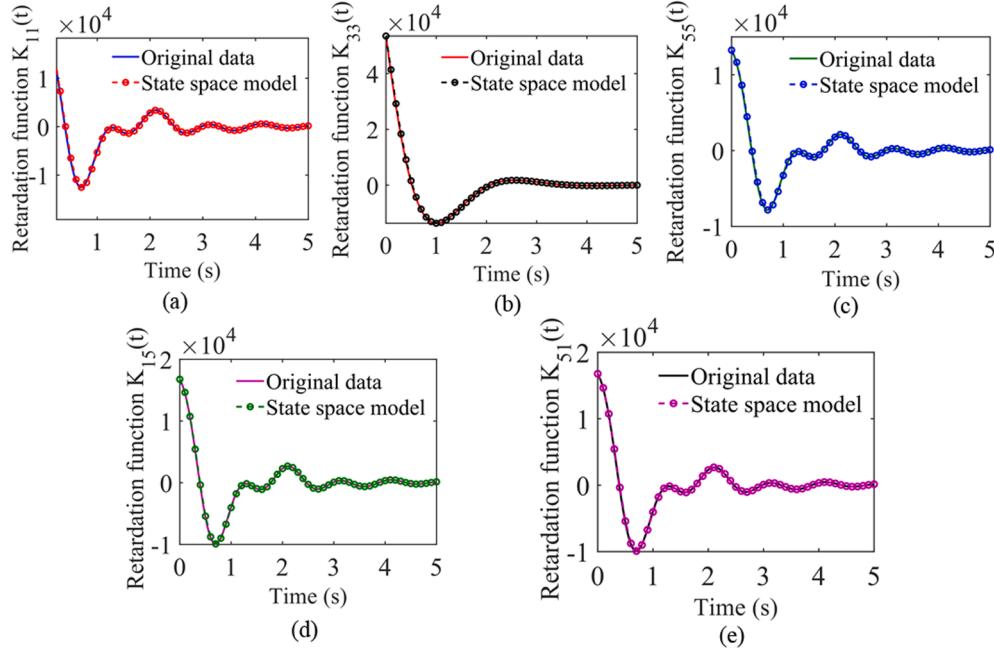


Fig. A3. Identification results for retardation function: (a) K_{11} , (b) K_{33} , (c) K_{55} , (d) K_{15} and (e) K_{51} .

Appendix A2. Derivation process of theoretical modeling equations

1) The derivation process of Eq. (15)

Based the decoupled mechanism for the proposed WEC, by using unit vectors $[1 \ 0 \ 0]^T$, $[0 \ 1 \ 0]^T$ and $[0 \ 0 \ 1]^T$ to denote the positive directions of surge (x-axis), sway (y-axis), and heave (z-axis), respectively, we can express $F_{pto,1}$, $F_{pto,2}$, and $F_{pto,3}$ as

$$F_{pto,1} = f_{pto,1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, F_{pto,2} = f_{pto,2} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, F_{pto,3} = f_{pto,3} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (15a)$$

where, $f_{pto,1}$, $f_{pto,2}$ and $f_{pto,3}$ represent the magnitudes of the respective PTO forces. A positive value indicates that the force acts along the positive axis, whereas a negative value indicates that it acts along the negative axis. Furthermore, combined with the two-argument property of the Kronecker delta function [77,78], we can simplify Eq. (15a) as follows:

$$F_{pto,r} = f_{pto,r} \begin{bmatrix} \delta_{1r} \\ \delta_{2r} \\ \delta_{3r} \end{bmatrix}, (r = 1, 2, 3) \quad (15b)$$

Thus, the key to deriving $F_{pto,r}$ lies in determining $f_{pto,r}$ based on the specific PTO structure. Since all of our translation PTOs share the same design, we can represent $f_{pto,r}$ with a unified expression. As illustrated in Fig. 18(a), each translational PTO includes a tension spring, a ball screw, a coupler, and a permanent magnet synchronous generator (PMSG). In the power generation process, the tension spring provides the PTO's restoring stiffness, the ball screw converts the floating body's translational motion into rotation, and this rotation is transmitted through the coupler to drive the PMSG rotor, which cuts magnetic flux lines to generate electricity. Therefore, the PTO force comprises the spring's restoring force, the moment required to drive the ball screw, coupler, and PMSG rotor, as well as the electromagnetic moment that overcomes the equivalent electromagnetic damping inside the PMSG.

According to Hooke's law, the spring's restoring force can be expressed as the product of the tension spring's stiffness $k_{T,r}$ and the corresponding PTO's displacement $x_{pto,r}$, as shown below

$$f_{spring,r} = k_{T,r} x_{pto,r} \quad (15c)$$

According to the relationship between the PMSG electromagnetic moment and its equivalent electromagnetic damping [33,79], we can obtain:

$$\tau_{PMSG,r} = c_{PMSG,r} \dot{\theta}_{g,r} \quad (15d)$$

where $\dot{\theta}_{g,r}$ is the rotational angular velocity of the PMSG in the translation PTO. $c_{PMSG,r}$ is the equivalent electromagnetic damping of the PMSG in the translation PTO, which corresponds to the PTO damping coefficient you were concerned about in your comment.

According to the relationship between rotational inertia and moment [33,48], we obtain the moment to drive the ball screw, coupler, and PMSG rotor.

$$\tau_{SCP,r} = (I_{S,r} + I_{C,r} + I_{P,r}) \ddot{\theta}_{g,r} \quad (15e)$$

where $\ddot{\theta}_{g,r}$ is the rotational angular acceleration of the PMSG in the translation PTO, which is equal to that of the ball screw and coupler due to the coaxial configuration of all three components. The $I_{S,r}$, $I_{C,r}$, and $I_{P,r}$ are the rotational inertia of the ball screw, coupler, and PMSG rotor for the corresponding translation PTO, respectively.

To convert the above moments ($\tau_{PMSG,r}$ and $\tau_{SCP,r}$) into the force at the contact interface between the PTO and the floating body, we use the basic force transmission relation in the ball screw [33,79], combined with Eqs. (15d) and (15e), to obtain

$$\begin{aligned} f_{troke,r} &= \frac{2\pi}{\lambda l_{lead}} (\tau_{PMSG,r} + \tau_{SCP,r}) \\ &= \frac{2\pi}{\lambda l_{lead}} \left(c_{PMSG,r} \dot{\theta}_{g,r} + (I_{S,r} + I_{C,r} + I_{P,r}) \ddot{\theta}_{g,r} \right) \end{aligned} \quad (15f)$$

where the l_{lead} represents the lead of the ball screw. λ denotes the transmission efficiency of the translation PTO.

Combined with Eqs. (15c) and (15f), we obtain the expression for $f_{pto,r}$ as follows

$$\begin{aligned} f_{pto,r} &= f_{spring,r} + f_{troke,r} \\ &= \frac{2\pi}{\lambda l_{lead}} \left(c_{PMSG,r} \dot{\theta}_{g,r} + (I_{S,r} + I_{C,r} + I_{P,r}) \ddot{\theta}_{g,r} \right) + k_{T,r} x_{pto,r} \end{aligned} \quad (15g)$$

According to the ball screw transmission principle, the rotation of the ball screw and the translational motion of the PTO are related by the following equation [49].

$$\begin{cases} \dot{\theta}_{g,r} = \frac{2\pi}{l_{lead}} \dot{x}_{pto,r} \\ \ddot{\theta}_{g,r} = \frac{2\pi}{l_{lead}} \ddot{x}_{pto,r} \end{cases} \quad (15h)$$

Substituting Eq. (15h) into Eq. (15g) and then into Eq. (15b), we can obtain the derivation of Eq. (15).

$$\mathbf{F}_{pto,r} = \left\{ \frac{2\pi}{\lambda l_{lead}} \left[c_{PMSG,r} \dot{x}_{pto,r} + (I_{S,r} + I_{C,r} + I_{P,r}) \frac{2\pi}{l_{lead}} \ddot{x}_{pto,r} \right] + k_{T,r} x_{pto,r} \right\} \begin{bmatrix} \delta_{1r} \\ \delta_{2r} \\ \delta_{3r} \end{bmatrix} \quad (15i)$$

2) The derivation process of Eq. (16)

Eq. (16) is used to describe the PTO moment $\mathbf{F}_{pto,l}$ acting on a floating body in rotational directions. Specifically, when $l = 4$ and $l = 5$, we have $\mathbf{F}_{pto,4}$ and $\mathbf{F}_{pto,5}$, which represent the PTO moment in roll and pitch, respectively. The decoupled design ensures that each PTO's operating axis aligns one-to-one with a specific degree of freedom of the floating body. This means that each PTO moment acts only in its corresponding direction and is zero in others.

Similar to the derivation process of above Eq. (15), we can use unit vectors $[1 \ 0 \ 0]^T$ and $[0 \ 1 \ 0]^T$ to denote the positive directions of roll (around the x-axis) and pitch (around the y-axis), respectively. Then, $\mathbf{F}_{pto,4}$ and $\mathbf{F}_{pto,5}$ can be expressed as

$$\mathbf{F}_{pto,4} = f_{pto,4} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{F}_{pto,5} = f_{pto,5} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (16a)$$

where, $f_{pto,1}$, $f_{pto,2}$ and $f_{pto,3}$ represent the magnitudes of the respective PTO moments. A positive value indicates that the moment acts around the positive axis, whereas a negative value indicates that it acts around the negative axis. Similarly, we use the Kronecker delta function to simplify Eq. (16a) as follows:

$$\mathbf{F}_{pto,l} = f_{pto,l} \begin{bmatrix} \delta_{4l} \\ \delta_{5l} \\ 0 \end{bmatrix}, (l = 4, 5) \quad (16b)$$

Thus, the key to deriving $\mathbf{F}_{pto,l}$ lies in determining $f_{pto,l}$ based on the specific PTO structure. As illustrated in Fig. 18(a), each rotation PTO includes a torsion spring, an epicyclic gearbox, a coupler, and a PMSG. In the power generation process, the torsion spring provides the PTO's restoring stiffness. The epicyclic gearbox amplifies the floating body's rotational motion and transmits the torque through the coupler to drive the PMSG rotor, which cuts magnetic flux lines to generate electricity. Therefore, this PTO moment comprises the torsion spring's restoring moment, the moment required to drive the epicyclic gearbox, coupler, and PMSG rotor, as well as the electromagnetic moment that overcomes the equivalent electromagnetic damping inside the PMSG.

Similar to the above approach for Eq. (15), we can obtain the torsion spring moment, PMSG electromagnetic moment, and the moment to drive the epicyclic gearbox, coupler, and PMSG rotor as follows

$$f_{spring,l} = k_{s,l}\theta_{pto,l} \quad (16c)$$

$$\tau_{PMSG,l} = c_{PMSG,l}\dot{\theta}_{g,l} \quad (16d)$$

$$\tau_{GCP,l} = (I_{G,l} + I_{C,l} + I_{P,l})\ddot{\theta}_{g,l} \quad (16e)$$

where the $k_{s,l}$ and $\theta_{pto,l}$ represent the torsion spring's stiffness and the corresponding PTO's angular displacement, respectively. $\dot{\theta}_{g,l}$ and $\ddot{\theta}_{g,l}$ are the rotational angular velocity and acceleration of the PMSG in the rotational PTO. Due to the coaxial configuration of all three components, these rotational angular velocity and acceleration values are equal to those of the epicyclic gearbox and coupler. $c_{PMSG,l}$ is the equivalent electromagnetic damping of the PMSG in the rotation PTO, which corresponds to the PTO damping coefficient you were concerned about in your comment. The $I_{G,l}$, $I_{C,l}$, and $I_{P,l}$ are the rotational inertia of the epicyclic gearbox, coupler, and PMSG rotor for the corresponding PTO, respectively.

Based on the fundamental force transmission relation in the epicyclic gearbox [68,69], we convert the $\tau_{PMSG,l}$ in Eq. (16d) and $\tau_{GCP,l}$ in Eq. (16e) into the moment acting at the contact interface between the rotation PTO and the floating body.

$$\begin{aligned} f_{troque,l} &= \frac{1+\alpha}{\eta} (\tau_{PMSG,l} + \tau_{GCP,l}) \\ &= \frac{1+\alpha}{\eta} \left(c_{PMSG,l}\dot{\theta}_{g,l} + (I_{G,r} + I_{C,r} + I_{P,r})\ddot{\theta}_{g,l} \right) \end{aligned} \quad (16f)$$

where the η denotes the transmission efficiency of the rotation PTO accounting for its operational friction, and α is the transmission ratio of epicyclic gearbox.

Combined with Eqs. (16c) and (16f), we obtain the expression for $f_{pto,l}$ as follows

$$\begin{aligned} f_{pto,l} &= f_{spring,l} + f_{troque,l} \\ &= \frac{1+\alpha}{\eta} \left(c_{PMSG,l}\dot{\theta}_{g,l} + (I_{G,r} + I_{C,r} + I_{P,r})\ddot{\theta}_{g,l} \right) + k_{s,l}\theta_{pto,l} \end{aligned} \quad (16g)$$

According to the ball screw transmission principle, the rotation of the ball screw and the rotational motion of the PTO are related by the following equation [68,69].

$$\begin{cases} \dot{\theta}_{g,l} = (1+\alpha)\dot{\theta}_{pto,l} \\ \ddot{\theta}_{g,l} = (1+\alpha)\ddot{\theta}_{pto,l} \end{cases} \quad (16h)$$

Substituting Eq. (16h) into Eq. (16g) and then into Eq. (16b), we can obtain the derivation of Eq. (16).

$$F_{pto,l} = \left\{ \frac{1+\alpha}{\eta} \left[c_{PMSG,l} \left(\dot{\theta}_{pto,l} + \alpha\dot{\theta}_{pto,l} \right) + (I_{G,l} + I_{C,l} + I_{P,l}) \left(\ddot{\theta}_{pto,l} + \alpha\ddot{\theta}_{pto,l} \right) \right] + k_{s,l}\theta_{pto,l} \right\} \begin{bmatrix} \delta_{4l} \\ \delta_{5l} \\ 0 \end{bmatrix} \quad (16i)$$

Data availability

Data will be made available on request.

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